Bridging GCSE and A-Level Maths

Welcome to A Level mathematics!

Congratulations on applying to study A Level Maths at Swakeleys School!

AS Maths has been designed to follow on from the GCSE in Maths, as a course for students who enjoy and are successful in Higher tier GCSE algebra topics. We would like to ensure that you understand which topics the course will cover, and give you the opportunity to practice the key skills that are required to have sooner or latter to take the Mathematics course.

The aim of this work is to ensure you are extremely fluent with your GCSE algebra skills so that you are able to access all aspects of the AS Maths course when it begins.

Remember to work on your maths little and often to keep it fresh in your mind.

Bridging GCSE and A-Level Maths

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Core Topics

- Indices
- Expanding brackets
- Factorising
- Surds
- Inequalities
- Coordinate Geometry
- Circle Theorems
- Algebraic Fractions
- Completing the Square
- Rearranging equations
- Solving simultaneous equations
- Drawing graphs
- Trigonometry – sine rule
- Trigonometry – cosine rule

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Indices

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The algebraic term $3x^5$ is written in **index form**. The 3 is called the **coefficient**. The x part of the term is called the **base**. The 5 is called the **power**, or **index**, or **exponent**. $3x^5$ means $3 \times x \times x \times x \times x \times x$

Indices follow some general rules.

Key point

Rule 1: Any number raised to the power zero is 1 $\quad x^0 = 1$

Rule 2: Negative powers may be written as reciprocals. $\quad x^{-n} = \dfrac{1}{x^n}$

Rule 3: Any base raised to the power of a unit fraction is a root. $\quad x^{\frac{1}{n}} = \sqrt[n]{x}$

Rule 1 has an exception when $x = 0$, as 0^0 is undefined.

$x^{\frac{1}{2}} = \sqrt{x}$ and $x^{\frac{1}{3}} = \sqrt[3]{x}$

You don't normally write the '2' in a square root.

You can combine terms in index form by following this simple set of rules called the **index laws**.

To use the index laws, the bases of all the terms must be the same.

Key point

Law 1: To multiply terms you add the indices. $\quad x^a \times x^b = x^{a+b}$

Law 2: To divide terms you subtract the indices. $\quad x^a \div x^b = x^{a-b}$

Law 3: To raise one term to another power you multiply the indices. $\quad (x^a)^b = x^{a \times b}$

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Indices

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Simplify the expressions in questions **1** to **44**. Show your working.

1 4^3

2 $(-3)^5$

3 $7^8 \div 7^4$

4 $c^7 \times c^4$

5 $(-p^3)^4$

6 $-(p^3)^4$

7 $(2c^{-3})^6$

8 $d^7 \times d^3 \times d^4$

9 $2e^3 \times 5e^4 \times 7e^2$

10 $4f^2 \times -3f^4 \times 9f^6$

11 $24g^{12} \div 6g^6$

12 $-44k^{44} \div 11k^{-11}$

13 $12f^2 \times 4f^4 \div 6f^3$

14 $12e^{13} \div 6e^4 \div 3e^7$

15 $3a \times 5b$

16 $5w \times 4x \times (-6x)$

17 $2d \times 3e \times 4f^2$

18 $3h^6 \times (-3h^8)$

19 $5r^5s^6 \times r^3s^4$

20 $5r^5s^6 \div r^3s^4$

21 $(g^2h^3) \times (-g^7h^5) \times (ghi^4)$

22 $(g^2h^3) \times (-g^7h^5) \div (ghi^4)$

23 $(-20z^9y^6) \div (-4z^4y)$

24 $\sqrt{36u^{36}}$

25 $(36u^{36})^{\frac{1}{2}}$

26 $\sqrt[3]{125t^{27}}$

27 $\sqrt[3]{-125t^{27}c^{12}}$

28 $(5)^{-1}$

29 $\left(\frac{1}{5}\right)^{-1}$

30 $6u^0$

31 $-50f^0$

32 $7y^0 - 4z^0$

33 4^{-2}

34 2^{-10}

35 $(3w)^{-2}$

36 $(3w^{-2})^{-2}$

37 $64^{\frac{3}{2}}$

38 $1024^{\frac{1}{5}}$

39 $1024^{\frac{4}{5}}$

40 $16^{\frac{3}{4}}$

41 $16^{\frac{-3}{4}}$

42 $\left(\frac{36}{49}\right)^{\frac{1}{2}}$

43 $\left(\frac{36}{49}\right)^{\frac{-1}{2}}$

44 $\left(\frac{36}{49}\right)^{\frac{3}{2}}$

45 If $5^n = 625$, find the value of n

46 If $3^m = 243$, find the value of m

47 If $6^{2t+1} = 216$, find the value of t

48 If $(2^{2b})(2^{-6b}) = 256$, find the value of b

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Indices - Answers

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1 64
2 -243
3 2401
4 c^{11}
5 p^{12}
6 $-p^{12}$
7 $64c^{-18}$
8 d^{14}
9 $70e^{9}$
10 $-108f^{12}$
11 $4g^{6}$
12 $-4k^{55}$
13 $8f^{3}$
14 $\frac{2}{3}e^{2}$
15 $15ab$
16 $-120wx^{2}$
17 $24def^{2}$
18 $-9h^{14}$
19 $5r^{8}s^{10}$
20 $5r^{2}s^{2}$
21 $-g^{10}h^{9}i^{4}$
22 $-g^{8}h^{7}i^{-4}$
23 $5z^{5}y^{5}$
24 $6u^{18}$
25 $6u^{18}$
26 $5t^{9}$
27 $-5t^{9}c^{4}$
28 $\frac{1}{5}$
29 5
30 6
31 -50
32 3
33 $\frac{1}{16}$
34 $\frac{1}{1024}$
35 $\frac{1}{9w^{2}}$
36 $\frac{w^{4}}{9}$
37 512
38 4
39 256
40 8
41 $\frac{1}{8}$
42 $\frac{6}{7}$
43 $\frac{7}{6}$
44 $\frac{216}{343}$
45 $n = 4$
46 $m = 5$
47 $t = 1$
48 $b = -2$

Bridging GCSE and A-Level

Expanding Brackets

Expand the following expressions.

1. $(2x + 3)(3x + 1)$
2. $(3y + 2)(4y + 3)$
3. $(3t + 1)(2t + 5)$
4. $(4t + 3)(2t - 1)$
5. $(5m + 2)(2m - 3)$
6. $(4k + 3)(3k - 5)$
7. $(3p - 2)(2p + 5)$
8. $(5w + 2)(2w + 3)$
9. $(2a - 3)(3a + 1)$
10. $(4r - 3)(2r - 1)$
11. $(3g - 2)(5g - 2)$
12. $(4d - 1)(3d + 2)$
13. $(5 + 2p)(3 + 4p)$
14. $(2 + 3t)(1 + 2t)$
15. $(4 + 3p)(2p + 1)$
16. $(6 + 5t)(1 - 2t)$
17. $(4 + 3n)(3 - 2n)$
18. $(2 + 3f)(2f - 3)$
19. $(3 - 2q)(4 + 5q)$
20. $(1 - 3p)(3 + 2p)$
21. $(4 - 2t)(3t + 1)$

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Answers

1 $6x^2 + 11x + 3$ **2** $12y^2 + 17y + 6$ **3** $6t^2 + 17t + 5$
4 $8t^2 + 2t - 3$ **5** $10m^2 - 11m - 6$ **6** $12k^2 - 11k - 15$
7 $6p^2 + 11p - 10$ **8** $10w^2 + 19w + 6$ **9** $6a^2 - 7a - 3$
10 $8r^2 - 10r + 3$ **11** $15g^2 - 16g + 4$ **12** $12d^2 + 5d - 2$
13 $8p^2 + 26p + 15$ **14** $6t^2 + 7t + 2$ **15** $6p^2 + 11p + 4$
16 $6 - 7t - 10t^2$ **17** $12 + n - 6n^2$ **18** $6f^2 - 5f - 6$
19 $12 + 7q - 10q^2$ **20** $3 - 7p - 6p^2$ **21** $4 + 10t - 6t^2$

Bridging GCSE and A-Level Maths

Expanding Brackets

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Try to spot the pattern in each of the following expressions so that you can immediately write down the expansion.

1. $(2x + 1)(2x - 1)$
2. $(3t + 2)(3t - 2)$
3. $(5y + 3)(5y - 3)$
4. $(4m + 3)(4m - 3)$
5. $(2k - 3)(2k + 3)$
6. $(4h - 1)(4h + 1)$
7. $(2 + 3x)(2 - 3x)$
8. $(5 + 2t)(5 - 2t)$
9. $(6 - 5y)(6 + 5y)$
10. $(a + b)(a - b)$
11. $(3t + k)(3t - k)$
12. $(2m - 3p)(2m + 3p)$
13. $(5k + g)(5k - g)$
14. $(ab + cd)(ab - cd)$
15. $(a^2 + b^2)(a^2 - b^2)$

1 $4x^2 - 1$
2 $9t^2 - 4$
3 $25y^2 - 9$
4 $16m^2 - 9$
5 $4k^2 - 9$
6 $16h^2 - 1$
7 $4 - 9x^2$
8 $25 - 4t^2$
9 $36 - 25y^2$
10 $a^2 - b^2$
11 $9t^2 - k^2$
12 $4m^2 - 9p^2$
13 $25k^2 - g^2$
14 $a^2b^2 - c^2d^2$
15 $a^4 - b^4$

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Expanding Brackets

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Expand the following squares.

HINTS AND TIPS

Remember *always* write down the bracket twice. Do not try to take any short cuts.

1. $(x + 5)^2$
2. $(m + 4)^2$
3. $(6 + t)^2$
4. $(3 + p)^2$
5. $(m - 3)^2$
6. $(t - 5)^2$
7. $(4 - m)^2$
8. $(7 - k)^2$
9. $(3x + 1)^2$
10. $(4t + 3)^2$
11. $(2 + 5y)^2$
12. $(3 + 2m)^2$
13. $(4t - 3)^2$
14. $(3x - 2)^2$
15. $(2 - 5t)^2$
16. $(6 - 5r)^2$
17. $(x + y)^2$
18. $(m - n)^2$
19. $(2t + y)^2$
20. $(m - 3n)^2$
21. $(x + 2)^2 - 4$
22. $(x - 5)^2 - 25$
23. $(x + 6)^2 - 36$
24. $(x - 2)^2 - 4$

1 $x^2 + 10x + 25$ **2** $m^2 + 8m + 16$ **3** $t^2 + 12t + 36$
4 $p^2 + 6p + 9$ **5** $m^2 - 6m + 9$ **6** $t^2 - 10t + 25$
7 $m^2 - 8m + 16$ **8** $k^2 - 14k + 49$ **9** $9x^2 + 6x + 1$
10 $16t^2 + 24t + 9$ **11** $25y^2 + 20y + 4$ **12** $4m^2 + 12m + 9$
13 $16t^2 - 24t + 9$ **14** $9x^2 - 12x + 4$ **15** $25t^2 - 20t + 4$
16 $25r^2 - 60r + 36$ **17** $x^2 + 2xy + y^2$ **18** $m^2 - 2mn + n^2$
19 $4t^2 + 4ty + y^2$ **20** $m^2 - 6mn + 9n^2$ **21** $x^2 + 4x$
22 $x^2 - 10x$ **23** $x^2 + 12x$ **24** $x^2 - 4x$

Bridging GCSE and A-Le

Factorising

Factorise the following.

1 $x^2 + 5x + 6$

2 $t^2 + 5t + 4$

3 $m^2 + 7m + 10$

4 $k^2 + 10k + 24$

5 $p^2 + 14p + 24$

6 $r^2 + 9r + 18$

7 $w^2 + 11w + 18$

8 $x^2 + 7x + 12$

9 $a^2 + 8a + 12$

10 $k^2 + 10k + 21$

11 $f^2 + 22f + 21$

12 $b^2 + 20b + 96$

13 $t^2 - 5t + 6$

14 $d^2 - 5d + 4$

15 $g^2 - 7g + 10$

16 $x^2 - 15x + 36$

17 $c^2 - 18c + 32$

18 $t^2 - 13t + 36$

19 $y^2 - 16y + 48$

20 $j^2 - 14j + 48$

21 $p^2 - 8p + 15$

22 $y^2 + 5y - 6$

23 $t^2 + 2t - 8$

24 $x^2 + 3x - 10$

25 $m^2 - 4m - 12$

26 $r^2 - 6r - 7$

27 $n^2 - 3n - 18$

28 $m^2 - 7m - 44$

29 $w^2 - 2w - 24$

30 $t^2 - t - 90$

31 $h^2 - h - 72$

32 $t^2 - 2t - 63$

33 $d^2 + 2d + 1$

34 $y^2 + 20y + 100$

35 $t^2 - 8t + 16$

36 $m^2 - 18m + 81$

37 $x^2 - 24x + 144$

38 $d^2 - d - 12$

39 $t^2 - t - 20$

40 $q^2 - q - 56$

HINTS AND TIPS

First decide on the signs in the brackets, then look at the numbers.

1 $(x + 2)(x + 3)$ **2** $(t + 1)(t + 4)$ **3** $(m + 2)(m + 5)$
4 $(k + 4)(k + 6)$ **5** $(p + 2)(p + 12)$ **6** $(r + 3)(r + 6)$
7 $(w + 2)(w + 9)$ **8** $(x + 3)(x + 4)$ **9** $(a + 2)(a + 6)$
10 $(k + 3)(k + 7)$ **11** $(f + 1)(f + 21)$ **12** $(b + 8)(b + 12)$
13 $(t - 2)(t - 3)$ **14** $(d - 4)(d - 1)$ **15** $(g - 2)(g - 5)$
16 $(x - 3)(x - 12)$ **17** $(c - 2)(c - 16)$ **18** $(t - 4)(t - 9)$
19 $(y - 4)(y - 12)$ **20** $(j - 6)(j - 8)$ **21** $(p - 3)(p - 5)$
22 $(y + 6)(y - 1)$ **23** $(t + 4)(t - 2)$ **24** $(x + 5)(x - 2)$
25 $(m + 2)(m - 6)$ **26** $(r + 1)(r - 7)$ **27** $(n + 3)(n - 6)$
28 $(m + 4)(m - 11)$ **29** $(w + 4)(w - 6)$ **30** $(t + 9)(t - 10)$
31 $(h + 8)(h - 9)$ **32** $(t + 7)(t - 9)$ **33** $(d + 1)^2$
34 $(y + 10)^2$ **35** $(t - 4)^2$ **36** $(m - 9)^2$
37 $(x - 12)^2$ **38** $(d + 3)(d - 4)$ **39** $(t + 4)(t - 5)$
40 $(q + 7)(q - 8)$

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Factorising

Each of these is the difference of two squares. Factorise them.

HINTS AND TIPS

Learn how to spot the difference of two squares as it occurs a lot in GCSE examinations.

1. $x^2 - 9$
2. $t^2 - 25$
3. $m^2 - 16$
4. $9 - x^2$
5. $49 - t^2$
6. $k^2 - 100$
7. $4 - y^2$
8. $x^2 - 64$
9. $t^2 - 81$
10. $x^2 - y^2$
11. $x^2 - 4y^2$
12. $x^2 - 9y^2$
13. $9x^2 - 1$
14. $16x^2 - 9$
15. $25x^2 - 64$
16. $4x^2 - 9y^2$
17. $9t^2 - 4w^2$
18. $16y^2 - 25x^2$

1 $(x + 3)(x - 3)$ **2** $(t + 5)(t - 5)$ **3** $(m + 4)(m - 4)$
4 $(3 + x)(3 - x)$ **5** $(7 + t)(7 - t)$ **6** $(k + 10)(k - 10)$
7 $(2 + y)(2 - y)$ **8** $(x + 8)(x - 8)$ **9** $(t + 9)(t - 9)$
10 $(x + y)(x - y)$ **11** $(x + 2y)(x - 2y)$ **12** $(x + 3y)(x - 3y)$
13 $(3x + 1)(3x - 1)$ **14** $(4x + 3)(4x - 3)$ **15** $(5x + 8)(5x - 8)$
16 $(2x + 3y)(2x - 3y)$ **17** $(3t + 2w)(3t - 2w)$ **18** $(4y + 5x)(4y - 5x)$

Bridging GCSE and A-Level Maths

Factorising

Factorise the following expressions.

1. $2x^2 + 5x + 2$
2. $7x^2 + 8x + 1$
3. $4x^2 + 3x - 7$
4. $24t^2 + 19t + 2$
5. $15t^2 + 2t - 1$
6. $16x^2 - 8x + 1$
7. $6y^2 + 33y - 63$
8. $4y^2 + 8y - 96$
9. $8x^2 + 10x - 3$
10. $6t^2 + 13t + 5$
11. $3x^2 - 16x - 12$
12. $7x^2 - 37x + 10$

1 $(2x + 1)(x + 2)$ **2** $(7x + 1)(x + 1)$ **3** $(4x + 7)(x - 1)$
4 $(3t + 2)(8t + 1)$ **5** $(3t + 1)(5t - 1)$ **6** $(4x - 1)^2$
7 $3(y + 7)(2y - 3)$ **8** $4(y + 6)(y - 4)$ **9** $(2x + 3)(4x - 1)$
10 $(2t + 1)(3t + 5)$ **11** $(x - 6)(3x + 2)$ **12** $(x - 5)(7x - 2)$

Bridging GCSE and A-Level Maths

Surds

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Key point

A **rational number** is one that you can write exactly in the form

$$\frac{p}{q}$$

where p and q are integers, $q \neq 0$

Numbers that you cannot write exactly in this form are **irrational numbers**. If you express them as decimals, they have an infinite number of non-repeating decimal places.

Some roots of numbers are irrational, for example, $\sqrt{3} = 1.732...$ and $\sqrt[3]{10} = 2.15443...$ are irrational numbers.

Key point

Irrational numbers involving roots, $\sqrt[n]{\ }$ or $\sqrt{\ }$, are called **surds**.

You can use the following laws to simplify surds

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

You usually write surds in their simplest form, with the smallest possible number written inside the root sign.

You can simplify surds by looking at their factors.

You should look for factors that are square numbers.

For example $\sqrt{80} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$

If $\sqrt{a}$ and $\sqrt{b}$ cannot be simplified, then you cannot simplify $\sqrt{a} + \sqrt{b}$ or $\sqrt{a} - \sqrt{b}$ for $a \neq b$

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Surds

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Calculations are often more difficult if surds appear in the denominator. You can simplify such expressions by removing any surds from the denominator. To do this, you multiply the numerator and denominator by the same value to find an **equivalent fraction** with surds in the numerator only. This is easier to simplify.

This process is called **rationalising the denominator**.

Key point

If the fraction is in the form

$\frac{k}{\sqrt{a}}$, multiply numerator and denominator by $\sqrt{a}$

$\frac{k}{a \pm \sqrt{b}}$, multiply numerator and denominator by $a \mp \sqrt{b}$

$\frac{k}{\sqrt{a} \pm \sqrt{b}}$, multiply numerator and denominator by $\sqrt{a} \mp \sqrt{b}$

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Surds

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Complete this exercise without a calculator.

1 Classify these numbers as rational or irrational.

a $1+\sqrt{25}$ b π^2 c $4-\sqrt{3}$

d $\sqrt{21}$ e $\sqrt{169}$ f $\left(\sqrt{8}\right)^2$

g $\left(\sqrt{17}\right)^3$

2 For each of these expressions, show that they can be written in the form $a\sqrt{b}$ where a and b are integers.

a $\sqrt{4}\times\sqrt{21}$ b $\sqrt{8}\times\sqrt{7}$

c $\sqrt{75}$ d $\sqrt{27}$

e $\dfrac{\sqrt{800}}{10}$ f $\left(\sqrt{8}\right)^3$

g $\left(\sqrt{17}\right)^3$ h $2\sqrt{3}\times3\sqrt{2}$

i $5\sqrt{6}\times7\sqrt{18}$ j $4\sqrt{24}\times6\sqrt{30}$

3 Show that these expressions can be expressed as positive integers.

a $\dfrac{\sqrt{128}}{\sqrt{2}}$ b $\dfrac{\sqrt{125}}{\sqrt{5}}$

4 Show that these expressions can be written in the form $\dfrac{a}{b}$, where a and b are positive integers.

a $\dfrac{\sqrt{32}}{\sqrt{200}}$ b $\dfrac{\sqrt{50}}{\sqrt{72}}$

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Surds

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5 Show that these expressions can be written in the form $a\sqrt{b}$, where a and b are integers.

a $\sqrt{54}$ **b** $\sqrt{432}$

c $\sqrt{1280}$ **d** $\sqrt{3388}$

e $\sqrt{2} \times \sqrt{20}$ **f** $\sqrt{2} \times \sqrt{126}$

g $\sqrt{20} + \sqrt{5}$ **h** $\sqrt{18} - \sqrt{2}$

i $\sqrt{150} - \sqrt{24}$ **j** $\sqrt{75} + \sqrt{12}$

k $\sqrt{27} - \sqrt{3}$ **l** $\sqrt{5} + \sqrt{45}$

m $\sqrt{363} - \sqrt{48}$ **n** $\sqrt{72} - \sqrt{288} + \sqrt{200}$

6 Show these expressions can be written in the form $a + b\sqrt{c}$, where a, b and c are integers.

a $\left(3\sqrt{6} + \sqrt{5}\right)^2$

b $\left(\sqrt{2}+3\right)\left(4+\sqrt{2}\right)$

c $\left(\sqrt{2}-3\right)\left(4-\sqrt{2}\right)$

d $\left(3\sqrt{5}+4\right)\left(2\sqrt{5}-6\right)$

e $\left(5\sqrt{3}+3\sqrt{2}\right)\left(4\sqrt{27}-5\sqrt{8}\right)$

7 Rationalise the denominators in these expressions and leave your answers in their simplest form. Show your working.

a $\frac{1}{\sqrt{13}}$ **b** $\frac{8}{\sqrt{6}}$

c $\frac{\sqrt{11}}{2\sqrt{5}}$ **d** $\frac{3}{\sqrt{2} - 1}$

e $\frac{3\sqrt{7} \times 5\sqrt{4}}{6\sqrt{7}}$ **f** $\frac{13\sqrt{15} - 2\sqrt{10}}{4\sqrt{75}}$

g $\frac{5}{8 - \sqrt{5}}$ **h** $\frac{2\sqrt{2}}{4+\sqrt{2}}$

i $\frac{\sqrt{6} - \sqrt{5}}{\sqrt{6} + \sqrt{5}}$ **j** $\frac{3\sqrt{11} - 4\sqrt{7}}{\sqrt{11} - \sqrt{7}}$

8 Rationalise the denominators and simplify these expressions. a, b and c are integers.

a $\frac{a+\sqrt{b}}{\sqrt{b}}$ **b** $\frac{a+\sqrt{b}}{a-\sqrt{b}}$

c $\frac{\sqrt{a}+b\sqrt{c}}{b\sqrt{c}}$ **d** $\frac{\sqrt{a}-b\sqrt{c}}{\sqrt{a}+\sqrt{b}}$

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Surds - Answers

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1 Parts **a, e** and **f** are rational; parts **b, c, d** and **g** are irrational

2 **a** $2\sqrt{21}$ **b** $2\sqrt{14}$ **c** $5\sqrt{3}$ **d** $3\sqrt{3}$
e $2\sqrt{2}$ **f** $16\sqrt{2}$ **g** $17\sqrt{17}$ **h** $6\sqrt{6}$
i $210\sqrt{3}$ **j** $288\sqrt{5}$

3 **a** 8 **b** 5

4 **a** $\frac{4}{10}$ or 0.4 **b** $\frac{5}{6}$

5 **a** $3\sqrt{6}$ **b** $12\sqrt{3}$ **c** $16\sqrt{5}$ **d** $22\sqrt{7}$
e $2\sqrt{10}$ **f** $6\sqrt{7}$ **g** $3\sqrt{5}$ **h** $2\sqrt{2}$
i $3\sqrt{6}$ **j** $7\sqrt{3}$ **k** $2\sqrt{3}$ **l** $4\sqrt{5}$
m $7\sqrt{3}$ **n** $4\sqrt{2}$

6 **a** $59+6\sqrt{30}$ **b** $14+7\sqrt{2}$ **c** $-14+7\sqrt{2}$
d $6-10\sqrt{5}$ **e** $120-14\sqrt{6}$

7 **a** $\frac{\sqrt{13}}{13}$ **b** $\frac{4\sqrt{6}}{3}$
c $\frac{\sqrt{55}}{10}$ **d** $3(\sqrt{2}+1)$
e 5 **f** $\frac{13\sqrt{5}}{20}-\frac{\sqrt{30}}{30}$
g $\frac{5(8+\sqrt{5})}{59}$ **h** $\frac{2(2\sqrt{2}-1)}{7}$
i $11-2\sqrt{30}$ **j** $\frac{5-\sqrt{77}}{4}$

8 **a** $\frac{a\sqrt{b}+b}{b}$ **b** $\frac{a^2+2a\sqrt{b}+b}{a^2-b}$
c $\frac{\sqrt{ac}+bc}{bc}$ **d** $\frac{a-\sqrt{ab}-b\sqrt{ac}+b\sqrt{bc}}{a-b}$

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Inequalities

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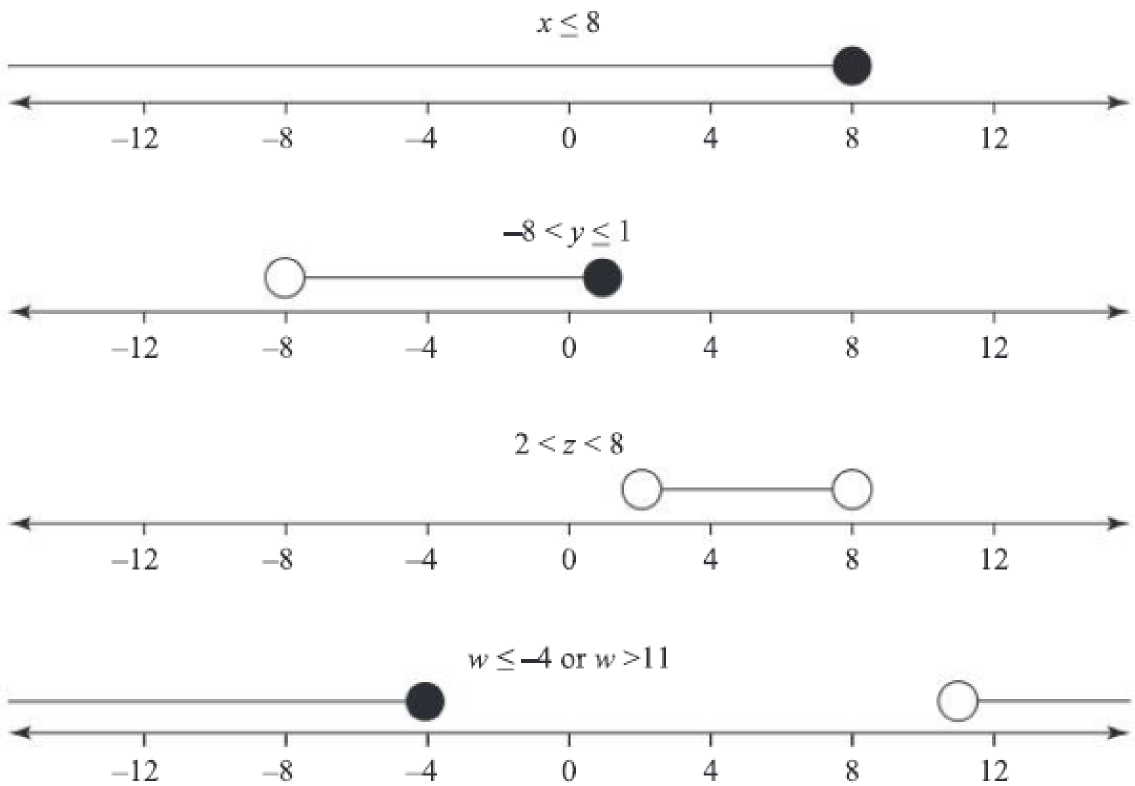
You can express **inequalities** using the symbols < (less than), > (greater than), ≤ (less than or equal to) and ≥ (greater than or equal to).

Key point

You can represent inequalities on a number line.

For example



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Inequalities

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On a number line, you use a dot, ●, when representing $\leq$ or $\geq$, and you use an empty circle, ○, when representing $<$ or $>$

You can also use set notation to represent inequalities.

For example, the last inequality could be represented in any of the following ways.

- $w \in \{w: w \leq -4 \text{ or } w > 11\}$
  w is an element of the set of values that are less than or equal to -4 or greater than 11
- $w \in \{w: w \leq -4\} \cup \{w: w > 11\}$
  w is an element of the union of two sets. This means w is in one set or the other.
- $w \in (-\infty, -4] \cup (11, \infty)$
  w is in the union of two intervals. Square brackets indicate the end value is included in the interval, round brackets indicate that the end value is not included in the interval.

To solve **linear inequalities** you follow the same rules for solving linear equations, but with one exception.

Key point

When you multiply or divide an inequality by a negative number, you reverse the inequality sign.

A **quadratic inequality** looks similar to a quadratic equation except it has an inequality sign instead of the '=' sign.

You can solve quadratic inequalities by starting the same way you would to solve quadratic equations. The answer, however, will be a range of values rather than up to two specific values.

Bridging GCSE and A-Level Maths

Inequalities

1 Show the following inequalities on a number line.

a $r > 7$ and $r \le 12$

b $s \ge 14$

c $3 < u \le 9$

d $v < 5$ and $v > 14$

2 Draw graphs to show these inequalities. You can check your sketches using a graphics calculator.

a $x > -4$

b $y \ge 5$

c $y + x < 6$

d $2y - 3x < 5$

e $3y + 4x \le 8$

f $2y > 10 - 4x$

g $y < x + 4;\ y + x + 1 > 0;\ x \le 5$

h $y \ge 2;\ \ x + y < 7;\ \ y - 2x - 4 \le 0$

3 Find the values of x for which

a $2x - 9 > -6$

b $15 - 2x \ge 8x + 34$

c $2(4x - 1) + 6 < 15 - 3x$

d $3(x - 3) + 6(5 - 4x) \le 54$

e $4(2x + 1) - 7(3x + 2) > 5(4 - 2x) - 6(3 - x)$

f $4\left(3x - \frac{1}{2}\right) + 2(8 - 3x) < 6\left(x + \frac{3}{2}\right) - 2\left(x - \frac{5}{2}\right)$

4 For each part **a** to **d**, sketch a suitable quadratic graph and use your sketch to solve the given inequality.

a $x^2 + x - 6 > 0$

b $x^2 + 11x + 28 < 0$

c $x^2 - 11x + 24 \le 0$

d $x^2 - 2x - 24 \ge 0$

Bridging GCSE and A-Level Maths

Inequalities

14/06/2023

5 Sketch graphs to solve each of these inequalities.

a $2x^2 - 3x - 2 > x + 4$

b $3x^2 + 19x - 14 < 2x - 8$

c $-3 + 13x - 4x^2 \leq 5x - 15$

d $6x^2 + 16x + 8 \geq 8 - 2x$

6 Complete the square or use the quadratic formula to solve these inequalities to 2 dp.

Sketch graphs to help you with these questions.

a $x^2 + 2x - 7 > 0$

b $x^2 + 7x + 8 < 0$

c $x^2 - 12x + 18 \leq 0$

d $x^2 - 3x - 21 \geq 0$

e $3x^2 - 5x - 7 > 0$

f $4x^2 + 17x - 4 < 0$

7 For each of the pairs of inequalities below

i Solve the inequalities simultaneously,

ii Record the points of intersection,

iii Shade the appropriate areas graphically.

a $y < 2x + 3;\ y > x^2$

b $x + y \leq 4;\ y > x^2 - 5x + 4$

c $y - 4x \leq 17;\ y \leq 4x^2 - 4x - 15;\ x \leq 4$

d $y - 2x - 20 < 0;\ y + 4x - 6 < 0;$
$y > x^2 - 5x - 24$

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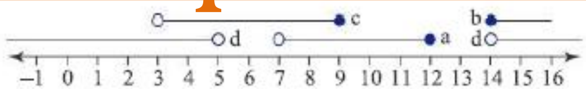
Inequalities - Answers

14/06/2023

1



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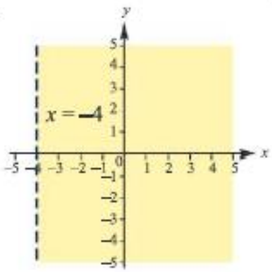

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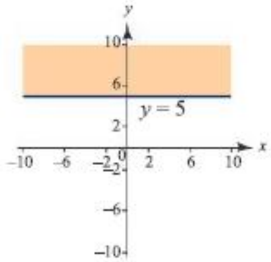

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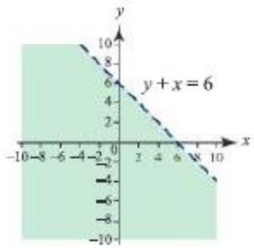

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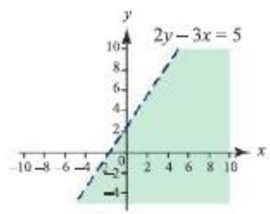

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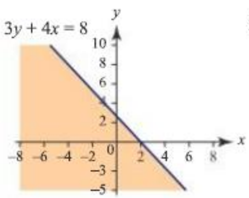

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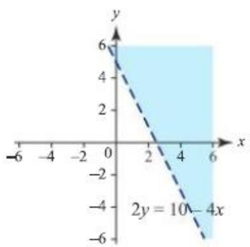

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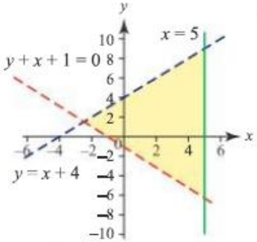

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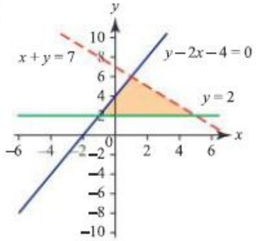


3 a $x > \frac{3}{2}$ b $x \le -1.9$ c $x < 1$

d $x \ge \frac{-11}{7}$ e $x < \frac{-4}{3}$ f $x < 0$

4 a $x < -3$ or $x > 2$ b $-7 < x < -4$

c $3 \le x \le 8$ d $x \le -4$ or $x \ge 6$

5 a $x > 3$ or $x < -1$ b $-6 < x < \frac{1}{3}$

c $x \le -1$ or $x \ge 3$ d $x \ge 0$ or $x \le -3$

6 a $x < -3.83$ or $x > 1.83$ b $-5.56 < x < -1.44$

c $1.76 \le x \le 10.24$ d $x < -3.32$ or $x > 6.32$

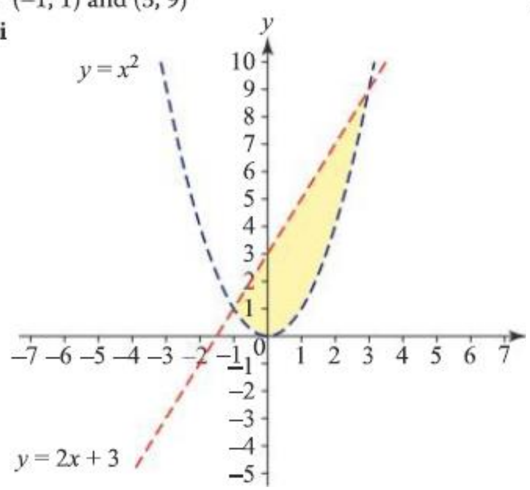
e $x < -0.91$ or $x > 2.57$ f $-4.47 < x < 0.22$

g $1.00 \le x \le 2.40$ h $x \le -0.38$ or $x \ge 3.05$

7 a i $y = 2x + 3$; $y = x^2$

ii $(-1, 1)$ and $(3, 9)$

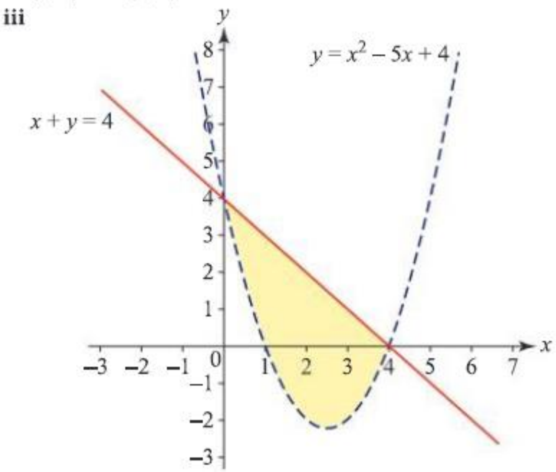
iii



b i $x + y = 4$; $y = x^2 - 5x + 4$

ii $(0, 4)$ and $(4, 0)$

iii



Bridging GCSE and A-Level Maths

Coordinate Geometry

The equation of a straight line can be written in the form $y = mx + c$

where m is the gradient and c is the y-intercept.

Key point

A straight line can also be written in the form

$y - y_1 = m(x - x_1)$

where (x_1, y_1) is a point on the line and m is the gradient.

You can rearrange the general equation of a straight line to get a formula for the gradient.

Key point

The gradient of a straight line through two points

(x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

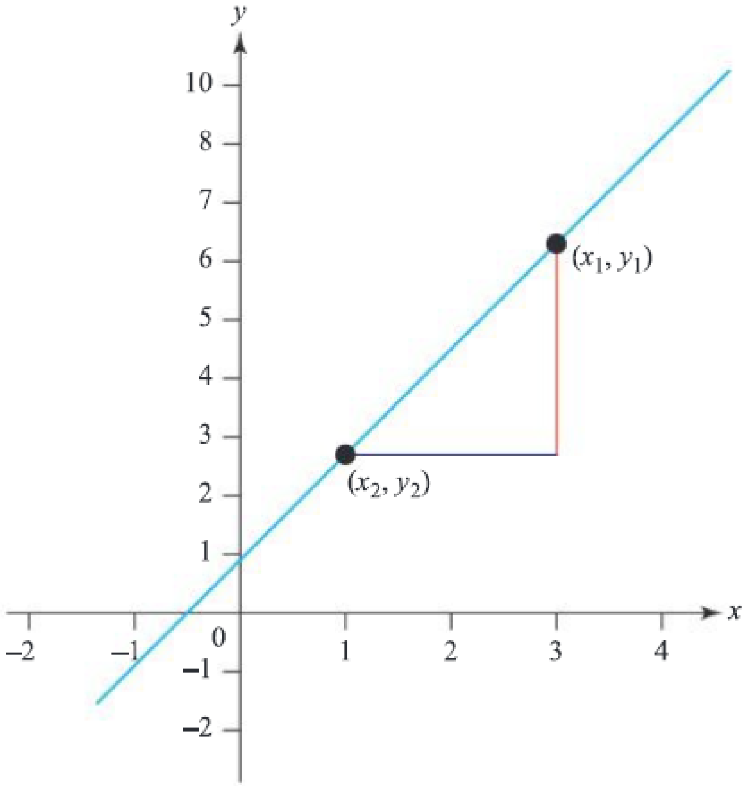
You can use Pythagoras' theorem to find the distance between two points.

Key point

The distance between two points (x_1, y_1) and (x_2, y_2) is given by the formula $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Bridging GCSE and A-Level Maths

Coordinate Geometry



Key point

The coordinates of the midpoint of the line joining (x_1, y_1) and (x_2, y_2) are given by the formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Bridging GCSE and A-Level Maths

Coordinate Geometry

You can use the gradients of two lines to decide if they are **parallel** or **perpendicular**.

Key point

Two lines are described by the equations

$y_1 = m_1x + c_1$ and $y_2 = m_2x + c_2$

If $m_1 = m_2$, the two lines are parallel.

If $m_1 \times m_2 = -1$, the two lines are perpendicular.

Bridging GCSE and A-Level Maths

Coordinate Geometry

3.1 Straight-line graphs

1 Find the gradient m and the y-intercept c of each of these lines.

a $y = 3x + 6$ **b** $y = 2 - 4x$

c $y = \frac{4x - 5}{2}$ **d** $y = -\frac{1}{3}(3 + 4x)$

Sketch, on separate diagrams, the lines with these equations. Label the points where the line crosses each axis with their coordinates.

2 By making y the subject of each equation, find the gradient m and the y-intercept c of these lines.

a $y - 2x + 1 = 0$ **b** $2y - 3x = 2$

c $4x - 3y = 1$ **d** $\frac{y}{4} + \frac{x}{2} = 3$

3 a Sketch, on the **same** diagram, the line $y - 2x = 1$ and the line $2y - 6x + 1 = 0$.

b Find the distance between the y-intercepts of these graphs.

4 a Sketch, on the **same** diagram, the line $y - 3x + 4 = 0$ and the line $3y + x = 6$.

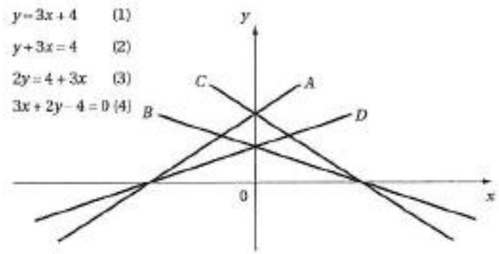
b Find the distance between the x-intercepts of these graphs.

5 Express the equations of these lines in the form $ay + bx + c = 0$, where a, b and c are integers.

a $y = -\frac{1}{2}x - \frac{3}{2}$ **b** $y = \frac{1}{3} - \frac{2x}{3}$

c $y = -\frac{3}{4}x + \frac{1}{2}$ **d** $y = \frac{2}{3}x - \frac{5}{2}$

6 The diagram shows a sketch of the lines A, B, C and D. The lines have equations (1), (2), (3) and (4).



Match each line with its equation.

7 Determine which of these equations describe a straight line. For those that do, sketch their graphs on separate diagrams.

a $2(2x - y) = 1$

b $y(x + 1) = 4 + xy$

c $\sqrt{x^2 + y^2} = 4$

d $\frac{x}{y} = 2$

e $\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$

f $3y(2x + 1) - 2x(3y + 2) = 6$

8 The line L has the same gradient as the line $4y + 2x + 3 = 0$.

a Find the gradient of L.

Given that this line L has the same y-intercept as the line $3y - 4x + 2 = 0$,

b find the y-intercept of L.

c Find the equation of L, giving your answer in the form $ay + bx + c = 0$ where a, b and c are integers.

9 The line L has equation $ay + bx = 10$, where a and b are constants.

The line crosses the y-axis at the point (0,5) and crosses the x-axis at the point (−2,0).

a Using this information, or otherwise, find the value of a and the value of b.

The point $P(4,q)$ lies on this line.

b Find the value of q.

Bridging GCSE and A-Level Maths

Coordinate Geometry

3.1 Straight-line graphs

1 Find the gradient m and the y-intercept c of each of these lines.

a $y = 3x + 6$ b $y = 2 - 4x$

c $y = \frac{4x - 5}{2}$ d $y = -\frac{1}{3}(3 + 4x)$

Sketch, on separate diagrams, the lines with these equations. Label the points where the line crosses each axis with their coordinates.

2 By making y the subject of each equation, find the gradient m and the y-intercept c of these lines.

a $y - 2x + 1 = 0$ b $2y - 3x = 2$

c $4x - 3y = 1$ d $\frac{y}{4} + \frac{x}{2} = 3$

3 a Sketch, on the **same** diagram, the line $y - 2x = 1$ and the line $2y - 6x + 1 = 0$.

b Find the distance between the y-intercepts of these graphs.

4 a Sketch, on the **same** diagram, the line $y - 3x + 4 = 0$ and the line $3y + x = 6$.

b Find the distance between the x-intercepts of these graphs.

5 Express the equations of these lines in the form $ay + bx + c = 0$, where a, b and c are integers.

a $y = -\frac{1}{2}x - \frac{3}{2}$ b $y = \frac{1}{3} - \frac{2x}{3}$

c $y = -\frac{3}{4}x + \frac{1}{2}$ d $y = \frac{2}{3}x - \frac{5}{2}$

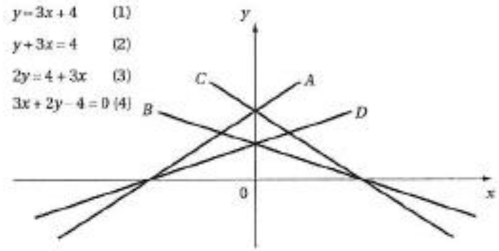
6 The diagram shows a sketch of the lines A, B, C and D. The lines have equations (1), (2), (3) and (4).

$y = 3x + 4$ (1)

$y + 3x = 4$ (2)

$2y = 4 + 3x$ (3)

$3x + 2y - 4 = 0$ (4)



Match each line with its equation.

7 Determine which of these equations describe a straight line. For those that do, sketch their graphs on separate diagrams.

a $2(2x - y) = 1$

b $y(x + 1) = 4 + xy$

c $\sqrt{x^2 + y^2} = 4$

d $\frac{x}{y} = 2$

e $\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$

f $3y(2x + 1) - 2x(3y + 2) = 6$

8 The line L has the same gradient as the line $4y + 2x + 3 = 0$.

a Find the gradient of L.

Given that this line L has the same y-intercept as the line $3y - 4x + 2 = 0$,

b find the y-intercept of L.

c Find the equation of L, giving your answer in the form $ay + bx + c = 0$ where a, b and c are integers.

9 The line L has equation $ay + bx = 10$, where a and b are constants.

The line crosses the y-axis at the point (0,5) and crosses the x-axis at the point (−2,0).

a Using this information, or otherwise, find the value of a and the value of b.

The point $P(4,q)$ lies on this line.

b Find the value of q.

Bridging GCSE and A-Level Maths

Coordinate Geometry

3.2 The equation of a line

1 Find the gradient of the lines which pass through these points.

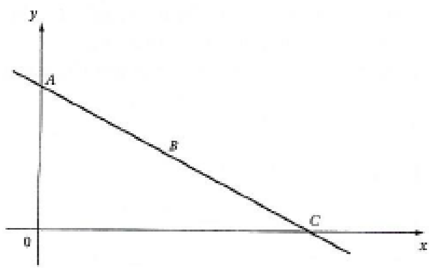
a $A(3,4)$ and $B(6,10)$

b $A(1,-3)$ and $B(3,7)$

c $A(-1,5)$ and $B(3,7)$

d $A(-4,-1)$ and $B(-6,5)$

2 The sketch shows the line L which passes through the points $A(0,3)$ and $B(2,2)$. This line crosses the x-axis at point C.



a Find the gradient of L.

b Find the equation of L. Give your answer in the form $ay + bx = c$ where a, b and c are integers.

c Find the coordinates of point C.

3 Find the equations of these lines.

a The line with gradient 4 which passes through the point $A(2,3)$.

b The horizontal line which passes through the point $C(35,-7)$.

c The line with gradient $-\frac{3}{2}$ which passes through the point $B(4,-2)$.

Give your answer for this line in the form $ay + bx = c$ where a, b and c are integers.

4 Find the equations of the lines passing through these points. Sketch each line on a separate diagram.

a $A(0,-1)$ and $B(5,14)$

b $A(2,5)$ and $B(4,1)$

c $A(-6,-4)$ and $B(10,8)$

d $A\left(\frac{1}{2},2\right)$ and $B(3,12)$

5 a A line passes through the points $A(1,5)$ and $B(3,p)$, where p is a constant. Given that the gradient of this line is 4, find the value of p.

b A line passes through the points $A(q,12)$ and $B(6,q)$, where q is a constant. Given that the gradient of this line is -7, find the value of q.

c A line passes through the points $E(r,r+1)$ and $F(8,0)$, where r is a constant.

Given that the gradient of this line is $-\frac{1}{2}$, find the value of r.

6 The line L passes through the points $P(-4,-3)$ and $Q(4,9)$. This line crosses the y-axis at point A and the x-axis at point B.

Handy hint
Sketch the line L.

a Find an equation for L.

b Write down the coordinates of A.

c Find the coordinates of B.

d Find the area of triangle OAB, where O is the origin.

7 A line passes through the points $S(3,-2)$ and $T(12,-14)$. This line crosses the y-axis at point A and the x-axis at point B.

Handy hint
Sketch the line.

a Find the coordinates of A and the coordinates of B.

b Show that the distance $AB = \frac{5}{2}$.

8 A line has equation $y = mx - 3$ where m is a constant. The point $A(-5,7)$ lies on this line.

a Find the value of m.

b Determine whether or not the point $B(-7,10)$ lies on this line.

9 The line L has equation $2y - 4x + k = 0$ where k is a constant.

The point $A\left(\frac{5}{2},\frac{1}{2}\right)$ lies on L.

a Show that $k = 9$.

b Find the y-intercept of L.

c Find the area of the triangle formed by this line and the two coordinate axes.

10 The line L passes through the points $P(1,4k)$ and $Q(k,4)$, where k is a constant, $k > 1$.

a Show that the gradient of L is -4.

b Find the equation of L, giving your answer in terms of k.

Point R is where L crosses the x-axis.

c Show that the coordinates of R are $(k+1,0)$.

d Find the value of k such that $PQ = QR$.

Bridging GCSE and A-Level Maths

Coordinate Geometry

3.3 Mid-points and distances

1 Find the coordinates of the mid-point of the line AB.

a $A(2,5)$, $B(10,3)$

b $A(5,-1)$, $B(-1,7)$

c $A(-6,11)$, $B(3,4)$

d $A\left(\frac{3}{2},\frac{5}{3}\right)$, $B\left(\frac{5}{2},\frac{1}{6}\right)$

2 For the points $A(3,2)$ and $B(p,q)$, where p and q are constants,

a find an expression for the coordinates of the mid-point of AB.

Given that this mid-point has coordinates $(4,1)$,

b find the value of p and the value of q.

3 Find, in terms of the constant k, the coordinates of the mid-point of AB. Simplify each answer as far as possible.

a $A(2,4)$, $B(k,2k)$

b $A(-2k,5)$, $B(0,2k+1)$

c $A(3k,2-k)$, $B(k,5k)$

4 The mid-point of AB is the point $C(2,3)$. If A has coordinates $(1,-2)$ find the coordinates of B.

5 Points $A(-4,4)$ and B are such that the mid-point of AB is the point $C(-3,7)$.

a Find the coordinates of B.

b Find the coordinates of the point D such that B is the mid-point of CD.

6 Points $A(p,3)$ and $B(14,q)$, where p and q are constants, are such that the mid-point of AB is the point $C(8,11)$. Point D is the mid-point of AC.

a Show that $p = 2$ and find the value of q.

b Find the coordinates of D.

It is given that the distance $AD = 5$.

c Find the distance DB.

7 The vertices of the square $ABCD$ have coordinates $A(1,3)$, $B(-3,7)$, $C(p,q)$ and $D(5,7)$, where p and q are constants.

a Find the coordinates of the mid-point of BD.

The diagonals of any square bisect each othe

b Using this information, or otherwise, find the coordinates of C.

8 Find the distance AB for these points. Give answers in simplified surd form where appropriate.

a $A(1,4)$, $B(6,2)$

b $A(2,-2)$, $B(5,7)$

c $A(-3,-1)$, $B(-5,9)$

d $A\left(-\frac{1}{2},\frac{3}{4}\right)$, $B\left(\frac{1}{2},\frac{3}{2}\right)$

9 Given the points $A(3,4)$, $B(6,-1)$ and $C(-2,7)$, prove that the triangle ABC is isosceles, but not equilateral.

10 A circle has diameter AB, where $A(4,-1)$ and $B(8,2)$.

a Find the coordinates of the centre of this circle.

b Show that the radius of this circle is $\frac{5}{2}$ units.

11 The point $C(2,-3)$ is the centre of a circle. The point $A(7,9)$ lies on this circle.

a Show that the radius of this circle is 13 units.

b Find the coordinates of the point B such that AB is a diameter of this circle.

Bridging GCSE and A-Level Maths

Coordinate Geometry

3.4 Parallel and perpendicular lines

1 Write down the gradient of any line which is

a parallel to the line $y = 5 - 3x$

b perpendicular to the line $y = 4x + 1$.

2 Find the gradient of any line which is

a parallel to the line $2y = 5x - 6$

b perpendicular to the line $3y + 4x = 1$.

3 By finding their gradients, show that these pairs of lines are parallel.

a $y = 2x - 4$
$y - 2x + 3 = 0$

b $2y - 3x = 1$
$y = \dfrac{4 + 3x}{2}$

c $2x + 4y - 3 = 0$
$2y + x + 1 = 0$

4 Show that these pairs of lines are perpendicular.

a $y = 3x + 4$
$3y + x = 3$

b $2y + 3x = 0$
$3y - 2x = 2$

c $y = \dfrac{11 - 5x}{3}$
$3x - 5y + 1 = 0$

5 The line L has equation $y = 2 - 4x$.

a Find the equation of the line which is parallel to L and which passes through the point (0,3).

b Find the equation of the line which is perpendicular to L and which has the same y-intercept as L.

6 The line L has equation $4y - 3x = 11$.

a Find the gradient of L.

b Find the equation of the line which is perpendicular to L and which passes through the point $A(6,-6)$.

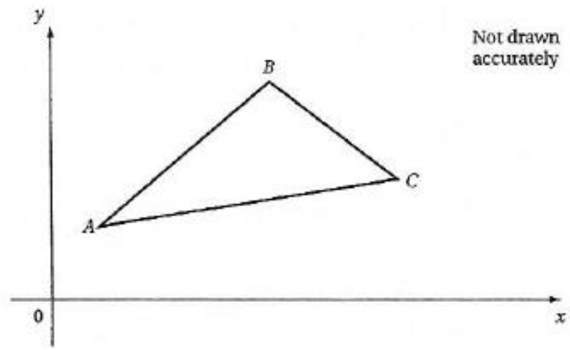
Give your answer in the form $ay + bx = c$ for integers a, b and c.

7 The line L has equation $y - 3x + 1 = 0$. The points $A(3,8)$ and $B(-1,k)$, where k is a constant, lie on L.

a Show that $k = -4$.

b Find the equation of the perpendicular bisector of AB. Give your answer in the form $ay + bx = c$, for integers a, b and c.

8 The diagram shows the triangle ABC where the vertices have coordinates $A(4,5)$, $B(8,10)$ and $C(13,6)$.



a Find the gradient of AB and the gradient of BC.

b Hence state the value of angle ABC.

c Show that triangle ABC is isosceles and hence state the value of angle BAC.

9 $A(2,7)$, $B(5,1)$ and $C(k,3)$, where k is a constant.

a Find the gradient of AB.

It is given that the line AB is perpendicular to the line BC.

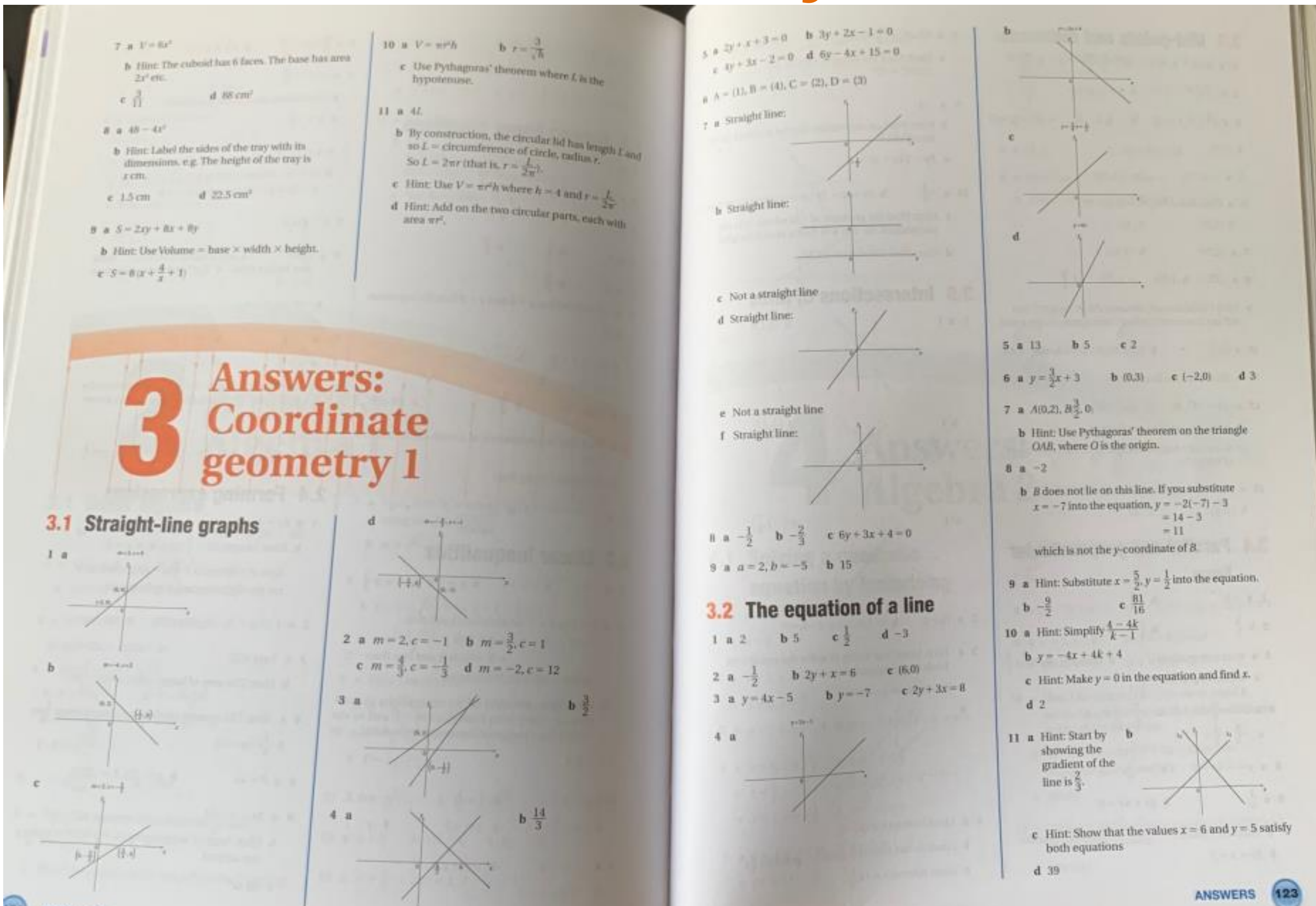
b Show that $k = 9$.

c Find the equation of the perpendicular bisector of the line AC.

Give your answer in the form $ay + bx + c = 0$ for integers a, b and c.

Bridging GCSE and A-Level Maths

Coordinate Geometry - Answers



7 a $V = 6x^2$

b Hint: The cuboid has 6 faces. The base has area $2x^2$ etc.

c $\frac{3}{11}$ d 88 cm^2

8 a $48 - 4x^2$

b Hint: Label the sides of the tray with its dimensions, e.g. The height of the tray is x cm.

c 1.5 cm d 22.5 cm^2

9 a $S = 2xy + 8x + 8y$

b Hint: Use Volume = base × width × height.

c $S = 8(x + \frac{4}{x} + 1)$

10 a $V = \pi r^2 h$ b $r = \frac{3}{\sqrt{h}}$

c Use Pythagoras' theorem where L is the hypotenuse.

11 a $4L$

b By construction, the circular lid has length L and so L = circumference of circle, radius r. So $L = 2\pi r$ (that is, $r = \frac{L}{2\pi}$).

c Hint: Use $V = \pi r^2 h$ where $h = 4$ and $r = \frac{L}{2\pi}$

d Hint: Add on the two circular parts, each with area πr^2.

3 Answers: Coordinate geometry 1

3.1 Straight-line graphs

1 a

b

c

d

2 a $m = 2, c = -1$ b $m = \frac{3}{2}, c = 1$

c $m = \frac{4}{3}, c = -\frac{1}{3}$ d $m = -2, c = 12$

3 a b $\frac{3}{2}$

4 a b $\frac{14}{3}$

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5 a $2y + x + 3 = 0$ b $3y + 2x - 1 = 0$

c $4y + 3x - 2 = 0$ d $6y - 4x + 15 = 0$

6 A = (1), B = (4), C = (2), D = (3)

7 a Straight line:

b Straight line:

c Not a straight line

d Straight line:

e Not a straight line

f Straight line:

8 a $-\frac{1}{2}$ b $-\frac{2}{3}$ c $6y + 3x + 4 = 0$

9 a $a = 2, b = -5$ b 15

3.2 The equation of a line

1 a 2 b 5 c $\frac{1}{2}$ d -3

2 a $-\frac{1}{2}$ b $2y + x = 6$ c (6,0)

3 a $y = 4x - 5$ b $y = -7$ c $2y + 3x = 8$

4 a

b

c

d

5 a 13 b 5 c 2

6 a $y = \frac{3}{2}x + 3$ b (0,3) c (−2,0) d 3

7 a $A(0,2)$, $B(\frac{3}{2}, 0)$

b Hint: Use Pythagoras' theorem on the triangle OAB, where O is the origin.

8 a -2

b B does not lie on this line. If you substitute $x = -7$ into the equation, $y = -2(-7) - 3 = 14 - 3 = 11$ which is not the y-coordinate of B.

9 a Hint: Substitute $x = \frac{5}{2}$, $y = \frac{1}{2}$ into the equation.

b $-\frac{9}{2}$ c $\frac{81}{16}$

10 a Hint: Simplify $\frac{4 - 4k}{k - 1}$

b $y = -4x + 4k + 4$

c Hint: Make $y = 0$ in the equation and find x.

d 2

11 a Hint: Start by showing the gradient of the line is $\frac{2}{3}$. b

c Hint: Show that the values $x = 6$ and $y = 5$ satisfy both equations

d 39

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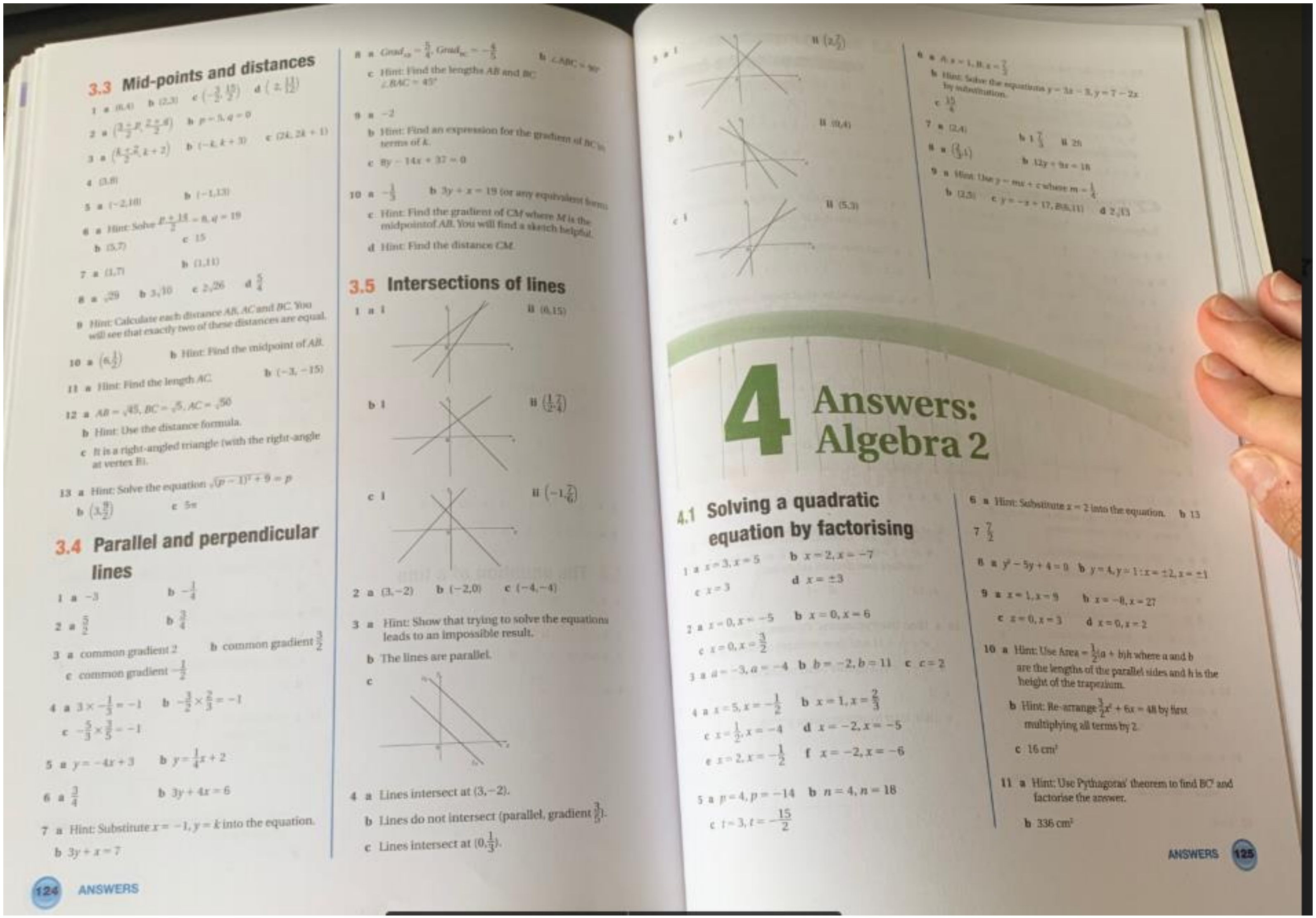
Bridging GCSE and A-Level Maths

Coordinate Geometry - Answers



3.3 Mid-points and distances

1 a (6,4) b (2,3) c $\left(-\frac{3}{2}, \frac{15}{2}\right)$

2 a $\left(\frac{3+p}{2}, \frac{2+q}{2}\right)$ b $p = 5, q = 0$

3 a $\left(\frac{k+2}{2}, k+2\right)$ b $(-k, k+3)$ c $(2k, 2k+1)$

4 (3,8)

5 a (−2,10) b (−1,13)

6 a Hint: Solve $\frac{p+15}{2} = 8$, $q = 19$
b (5,7) c 15

7 a (1,7) b (1,11)

8 a $\sqrt{29}$ b $3\sqrt{10}$ c $2\sqrt{26}$ d $\frac{5}{4}$

9 Hint: Calculate each distance AB, AC and BC. You will see that exactly two of these distances are equal.

10 a $\left(6, \frac{1}{2}\right)$ b Hint: Find the midpoint of AB.

11 a Hint: Find the length AC. b (−3, −15)

12 a $AB = \sqrt{45}, BC = \sqrt{5}, AC = \sqrt{50}$
b Hint: Use the distance formula.
c It is a right-angled triangle (with the right-angle at vertex B).

13 a Hint: Solve the equation $\sqrt{(p-1)^2 + 9} = p$
b $\left(3, \frac{9}{2}\right)$ c 5π

3.4 Parallel and perpendicular lines

1 a −3 b $-\frac{1}{4}$

2 a $\frac{5}{2}$ b $\frac{3}{4}$

3 a common gradient 2 b common gradient $\frac{3}{2}$
c common gradient $-\frac{1}{2}$

4 a $3 \times -\frac{1}{3} = -1$ b $-\frac{3}{2} \times \frac{2}{3} = -1$
c $-\frac{5}{3} \times \frac{3}{5} = -1$

5 a $y = -4x + 3$ b $y = \frac{1}{4}x + 2$

6 a $\frac{3}{4}$ b $3y + 4x = 6$

7 a Hint: Substitute $x = -1, y = k$ into the equation.
b $3y + x = 7$

8 a $Grad_{AB} = \frac{5}{4}, Grad_{BC} = -\frac{4}{5}$ b ∠ABC = 90°
c Hint: Find the lengths AB and BC. ∠BAC = 45°

9 a −2
b Hint: Find an expression for the gradient of BC in terms of k.
c $8y - 14x + 37 = 0$

10 a $-\frac{1}{3}$ b $3y + x = 19$ (or any equivalent form)
c Hint: Find the gradient of CM where M is the midpoint of AB. You will find a sketch helpful.
d Hint: Find the distance CM.

3.5 Intersections of lines

1 a i ii (0,15)
b i ii $\left(\frac{1}{2}, \frac{7}{4}\right)$
c i ii $\left(-1, \frac{7}{6}\right)$

2 a (3,−2) b (−2,0) c (−4,−4)

3 a Hint: Show that trying to solve the equations leads to an impossible result.
b The lines are parallel.
c

4 a Lines intersect at (3,−2).
b Lines do not intersect (parallel, gradient $\frac{3}{5}$).
c Lines intersect at $\left(0, \frac{1}{3}\right)$.

5 a i ii $\left(2, \frac{7}{2}\right)$
b i ii (0,4)
c i ii (5,3)

6 a $A: x = 1, B: x = \frac{7}{3}$
b Hint: Solve the equations $y = 3x - 3, y = 7 - 2x$ by substitution.
c $\frac{15}{4}$

7 a (2,4) b $1\frac{7}{3}$ ii 2

8 a $\left(\frac{2}{3}, 1\right)$ b $12y + 9x = 18$

9 a Hint: Use $y = mx + c$ where $m = \frac{1}{4}$.
b (2,5) c $y = -x + 17$, B(6,11) d $2\sqrt{13}$

4 Answers: Algebra 2

4.1 Solving a quadratic equation by factorising

1 a $x = 3, x = 5$ b $x = 2, x = -7$
c $x = 3$ d $x = \pm 3$

2 a $x = 0, x = -5$ b $x = 0, x = 6$
c $x = 0, x = \frac{3}{2}$

3 a $a = -3, a = -4$ b $b = -2, b = 11$ c $c = 2$

4 a $x = 5, x = -\frac{1}{2}$ b $x = 1, x = \frac{2}{3}$
c $x = \frac{1}{2}, x = -4$ d $x = -2, x = -5$
e $x = 2, x = -\frac{1}{2}$ f $x = -2, x = -6$

5 a $p = 4, p = -14$ b $n = 4, n = 18$
c $t = 3, t = -\frac{15}{2}$

6 a Hint: Substitute $x = 2$ into the equation. b 13

7 $\frac{7}{2}$

8 a $y^2 - 5y + 4 = 0$ b $y = 4, y = 1 : x = \pm 2, x = \pm 1$

9 a $x = 1, x = 9$ b $x = -8, x = 27$
c $x = 0, x = 3$ d $x = 0, x = 2$

10 a Hint: Use Area $= \frac{1}{2}(a + b)h$ where a and b are the lengths of the parallel sides and h is the height of the trapezium.
b Hint: Re-arrange $\frac{3}{2}x^2 + 6x = 48$ by first multiplying all terms by 2.
c 16 cm²

11 a Hint: Use Pythagoras' theorem to find BC² and factorise the answer.
b 336 cm²

Bridging GCSE and A-Level Maths

Coordinate Geometry

1 **a** Write down the equation of the straight line with gradient $-\frac{2}{3}$ that passes through the point $(-4, 7)$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

b Does the point $(13, 3)$ lie on the line described in part **a**?

2 Find the gradient and y-intercept of the line $4x - 3y = 8$

3 **a** Show that the lines $2x - 3y = 4$ and $6x + 4y = 7$ are perpendicular.

b Show that the lines $2x - 3y = 4$ and $8x - 12y = 7$ are parallel.

4 Write down the gradient and y-intercept of the line $\frac{2}{3}x + \frac{3}{4}y + \frac{7}{8} = 0$

5 Calculate the gradient of the straight-line segment joining the points $(-5, -6)$ and $(4, -1)$

Hence write down the equation of the line.

6 Write, in both the form $y = mx + c$ and the form $ax + by + c = 0$, the equation of the line with gradient -3 passing through $(-8, -1)$

7 Work out the midpoint and length of the line segment joining each of these pairs of points.

a $(2, 2)$ and $(6, 10)$

b $(-3, -4)$ and $(2, -3)$

c $(0, 0)$ and $(\sqrt{5}, 2\sqrt{3})$

8 Which of these lines are parallel or perpendicular to each other?

$2x + 3y = 4$ $\quad$ $4x - 5y = 6$ $\quad$ $y = 4x + 8$

$10x - 8y = 5$ $\quad$ $10x + 8y = 5$ $\quad$ $3y - 12x = 7$

$6x + 9y = 12$

9 **a** Write down the equation of the straight line through the point $(5, -4)$ which is parallel to the line $2x + 3y - 6 = 0$

b Write down the equation of the straight line through the point $(-2, -3)$ which is perpendicular to the line $3x + 6y + 5 = 0$

Bridging GCSE and A-Level Maths

Coordinate Geometry - Answers

1 **a** $2x + 3y - 13 = 0$ **b** No.

2 The gradient is $\frac{4}{3}$ and the y-intercept is $-\frac{8}{3}$

3 **a** The gradient of $2x - 3y = 4$ is $\frac{2}{3}$, the gradient of $6x + 4y = 7$ is $\frac{-6}{4} = \frac{-3}{2}$, and $\frac{2}{3} \times \frac{-3}{2} = -1$ so the lines are perpendicular.

b The gradient of $2x - 3y = 4$ is $\frac{2}{3}$ and the gradient of $8x - 12y = 7$ is $\frac{8}{12} = \frac{2}{3}$

Both lines have the same gradient, so the lines are parallel.

4 Gradient $= \frac{-8}{9}$ and y-intercept $= -\frac{7}{6}$

5 $m = \frac{-1 - (-6)}{4 - -5} = \frac{5}{9}$ and $9y - 5x + 29 = 0$

6 $y = -3x - 25$ or $y + 3x + 25 = 0$

7 **a** $(4, 6)$, length $4\sqrt{5}$ **b** $\left(\frac{-1}{2}, \frac{-7}{2}\right)$, length $\sqrt{26}$

c $\left(\frac{\sqrt{5}}{2}, \frac{2\sqrt{3}}{2}\right)$, length $\sqrt{17}$

8 $y = 4x + 8$ and $3y - 12x = 7$ are parallel; $2x + 3y = 4$ and $6x + 9y = 12$ are parallel; $4x - 5y = 6$ and $10x + 8y = 5$ are perpendicular.

9 **a** $3y + 2x + 2 = 0$ **b** $y - 2x - 1 = 0$

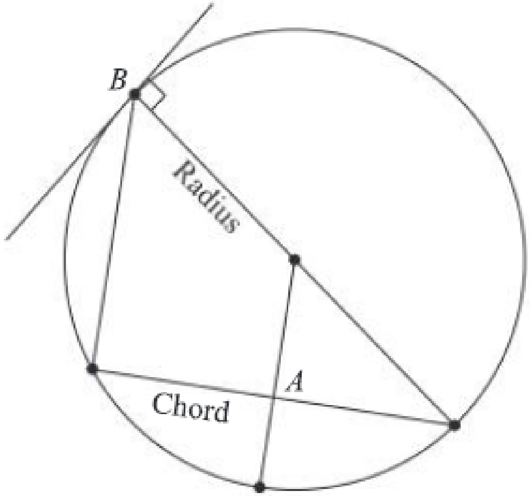
Bridging GCSE and A-Level Maths

Circle Theorems

When you're working with equations of circles, it's useful to remember some facts about the lines and angles in a circle. You should have come across these before in your studies.

Key point

- If a triangle passes through the centre of the circle, and all three corners touch the circumference of the circle, then the triangle is right-angled.
- The perpendicular line from the centre of the circle to a chord bisects the chord (Point *A* in the diagram).
- Any tangent to a circle is perpendicular to the radius at the point of contact (*B*).

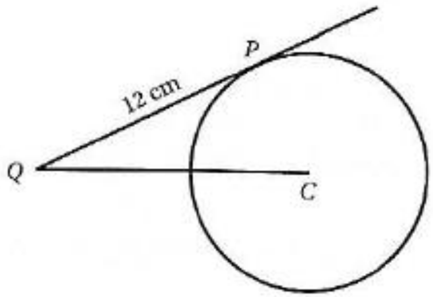


Bridging GCSE and A-Level Maths

Circle Theorems

5.4 Three circle theorems

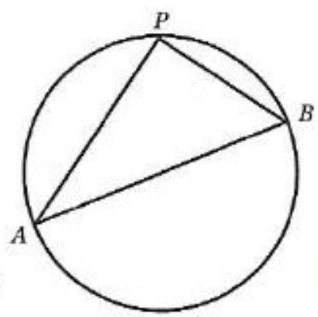
1 The diagram shows a circle with centre C and radius 5 cm. A tangent to the circle at point P is also shown. Q is a point on this tangent such that the distance $PQ = 12$ cm



a Show that the distance $QC = 13$ cm.

b Determine whether the mid-point M of QC lies inside or outside this circle.

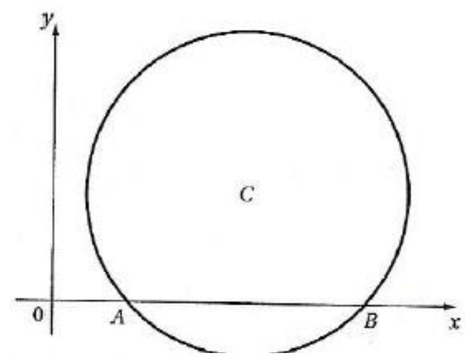
2 The diagram shows a circle with radius 10 cm. The line AB is a diameter of this circle. P is a point on this circle such that the distance $AP = 16$ cm.



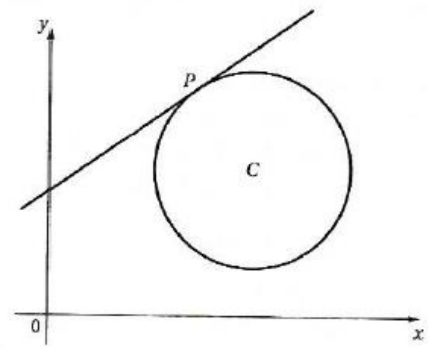
a Show that the distance $BP = 12$ cm.

b Find the area of triangle ABP.

3 The diagram shows a circle with centre C. The circle cuts the x-axis at points $A(2,0)$ and $B(8,0)$.



4 The diagram shows a circle with centre $C(5,3)$. Also shown is a tangent to the circle at point $P(2,9)$.



a Show that the gradient of the line CP is -2.

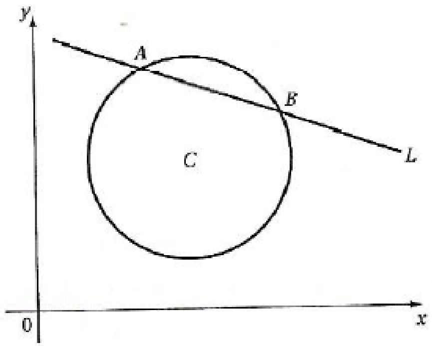
b Hence write down the gradient of this tangent.

c Find the equation of this tangent. Give your answer in the form $ay = bx + c$ where a, b and c are integers.

Bridging GCSE and A-Level Maths

Circle Theorems

5 The diagram shows a circle with centre $C(8,4)$. The line L has equation $3y + x = 6$ and intersects this circle at points A and B, as shown.

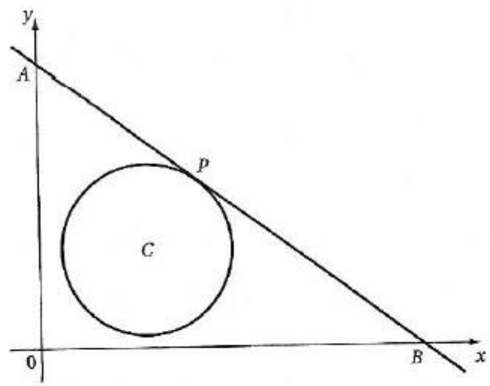


a Find the gradient of line L.

b Hence write down the gradient of the perpendicular bisector of AB.

c Find the equation of the perpendicular bisector of AB.

6 The diagram shows a circle with centre C(5,4). The tangent to this circle at the point P(8,6) has been drawn.



a Show that the gradient of this tangent is $-\frac{3}{2}$.

b Find the equation for this tangent. Give your answer in the form $ay + bx = c$ where a, b and c are integers.

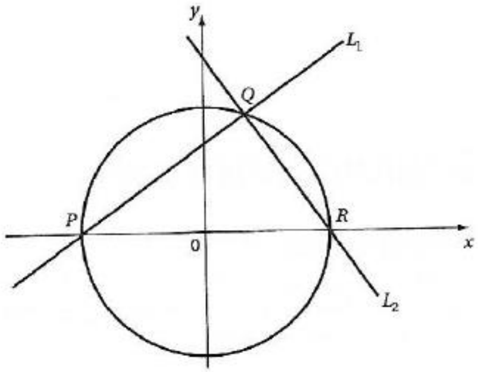
c Find the coordinates of the points A and B where this tangent crosses the x and y axes.

d Show that the region inside triangle OAB excluding this circle has area $(108 - 13\pi)$ squared units.

7 The diagram shows a circle, centred at the origin, and with radius 5.

The line L_1 intersects this circle at the point $P(-5,0)$ and the point Q.

The line L_2 intersects this circle at the point Q and the point $R(5,0)$.



The equation of L_1 is $2y = x + 5$.

a Show that the equation of L_2 is $y = 10 - 2x$.

b Find the coordinates of point Q.

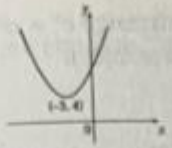
c By using the diameter PR, show that triangle PQR has area 20 square units.

d Hence, or otherwise, state the value of $QP \times QR$.

Bridging GCSE and A-Level Maths

Circle Theorems - Answers

7 a $(x+3)^2+4$

b 

c $k < 4$

8 a Hint: Any solution to $x^2 - 8x + 29 - k = 0$ corresponds to an intersection point of the curve $y = x^2 - 10x + 29$ and line $y = k - 2x$.

b Hint: $x^2 - 8x + 29 - k = (x-4)^2 + 13 - k$.

c (4,5)

9 a Hint: Rearrange $x^2 - 8x + 21 = -x^2 + 6x + 9$.

b $A(1,14)$, $B(6,9)$

c Hint: Start by finding the gradient and the mid-point of AB.

d $(-0.2,7.8)$, $(5.2,13.2)$

10 a $A(1,8)$, $B(7,20)$ b 36

5.4 Three circle theorems

1 a Hint: Use Pythagoras' theorem on the right-angled triangle QPC.

b M lies outside the circle.

2 a Hint: Use Pythagoras' theorem on the right-angled triangle APB.

b 96 cm²

3 a The perpendicular bisector of AB is the line $x = 5$ and this line must pass through the centre of the circle.

b $\sqrt{7}$

4 a Hint: Use the formula Gradient $= \frac{y_2 - y_1}{x_2 - x_1}$

b $\frac{1}{2}$

c $2y = x + 16$

5 a $-\frac{1}{3}$

b 3

c $y = 3x - 20$

6 a Hint: Start by calculating the gradient of PC.

b $2y + 3x = 36$

c $A(0,18)$, $B(12,0)$

d Hint: Start by finding the radius r of the circle by calculating the distance CP. The area of the circle is πr^2.

7 a Hint: Start by using the gradient of L_1 to work out the gradient of L_2.

b (3,4)

c Hint: Use the diameter as the base of the triangle.

d 40

Bridging GCSE and A-Level Maths

Algebraic Fractions

1 Simplify each of these.

a $\frac{x}{2} + \frac{x}{3}$

b $\frac{3x}{4} + \frac{x}{5}$

c $\frac{3x}{4} + \frac{2x}{5}$

d $\frac{x}{2} + \frac{y}{3}$

e $\frac{xy}{4} + \frac{2}{x}$

f $\frac{x+1}{2} + \frac{x+2}{3}$

g $\frac{2x-1}{2} + \frac{3x-1}{4}$

h $\frac{x}{5} + \frac{2x-1}{3}$

i $\frac{x-2}{2} + \frac{x+3}{4}$

j $\frac{x-4}{5} + \frac{2x-3}{2}$

2 Simplify each of these.

a $\frac{x}{2} - \frac{x}{3}$

b $\frac{3x}{4} - \frac{x}{5}$

c $\frac{3x}{4} - \frac{2x}{5}$

d $\frac{x}{2} - \frac{y}{3}$

e $\frac{xy}{4} - \frac{2}{y}$

f $\frac{x+1}{2} - \frac{x+2}{3}$

g $\frac{2x+1}{2} - \frac{3x+3}{4}$

h $\frac{x}{5} - \frac{2x+1}{3}$

i $\frac{x-2}{2} - \frac{x-3}{4}$

j $\frac{x-4}{5} - \frac{2x-3}{2}$

Bridging GCSE and A-Level Maths

Algebraic Fractions

3 Solve the following equations.

a $\frac{x+1}{2} + \frac{x+2}{5} = 3$

b $\frac{x+2}{4} + \frac{x+1}{7} = 3$

c $\frac{4x+1}{3} - \frac{x+2}{4} = 2$

d $\frac{2x-1}{3} + \frac{3x+1}{4} = 7$

e $\frac{2x+1}{2} - \frac{x+1}{7} = 1$

f $\frac{3x+1}{5} - \frac{5x-1}{7} = 0$

4 Simplify each of these.

a $\frac{x}{2} \times \frac{x}{3}$

b $\frac{2x}{7} \times \frac{3y}{4}$

c $\frac{4x}{3y} \times \frac{2y}{x}$

d $\frac{4y^2}{9x} \times \frac{3x^2}{2y}$

e $\frac{x}{2} \times \frac{x-2}{5}$

f $\frac{x-3}{15} \times \frac{5}{2x-6}$

g $\frac{2x+1}{2} \times \frac{3x+1}{4}$

h $\frac{x}{5} \times \frac{2x+1}{3}$

i $\frac{x-2}{2} \times \frac{4}{x-3}$

j $\frac{x-5}{10} \times \frac{5}{x^2-5x}$

Bridging GCSE and A-Level Maths

Algebraic Fractions

5 Simplify each of these.

a $\frac{x}{2} \div \frac{x}{3}$

b $\frac{2x}{7} \div \frac{4y}{14}$

c $\frac{4x}{3y} \div \frac{x}{2y}$

d $\frac{4y^2}{9x} \div \frac{2y}{3x^2}$

e $\frac{x}{2} \div \frac{x-2}{5}$

f $\frac{x-3}{15} \div \frac{5}{2x-6}$

g $\frac{2x+1}{2} \div \frac{4x+2}{4}$

h $\frac{x}{6} \div \frac{2x^2+x}{3}$

i $\frac{x-2}{12} \div \frac{4}{x-3}$

j $\frac{x-5}{10} \div \frac{x^2-5x}{5}$

6 Simplify each of these. Factorise and cancel where appropriate.

a $\frac{3x}{4} + \frac{x}{4}$

b $\frac{3x}{4} - \frac{x}{4}$

c $\frac{3x}{4} \times \frac{x}{4}$

d $\frac{3x}{4} \div \frac{x}{4}$

e $\frac{3x+1}{2} + \frac{x-2}{5}$

f $\frac{3x+1}{2} - \frac{x-2}{5}$

g $\frac{3x+1}{2} \times \frac{x-2}{5}$

h $\frac{x^2-9}{10} \times \frac{5}{x-3}$

i $\frac{2x+3}{5} \div \frac{6x+9}{10}$

j $\frac{2x^2}{9} - \frac{2y^2}{3}$

Bridging GCSE and A-Level Maths

Algebraic Fractions

7 Show that each algebraic fraction simplifies to the given expression.

a $\frac{2}{x+1} + \frac{5}{x+2} = 3$ simplifies to $3x^2 + 2x - 3 = 0$

b $\frac{4}{x-2} + \frac{7}{x+1} = 3$ simplifies to $3x^2 - 14x + 4 = 0$

c $\frac{3}{4x+1} - \frac{4}{x+2} = 2$ simplifies to $8x^2 + 31x + 2 = 0$

d $\frac{2}{2x-1} - \frac{6}{x+1} = 11$ simplifies to $22x^2 + 21x - 19 = 0$

e $\frac{3}{2x-1} - \frac{4}{3x-1} = 1$ simplifies to $x^2 - x = 0$

8 Solve the following equations.

a $\frac{4}{x+1} + \frac{5}{x+2} = 2$

b $\frac{18}{4x-1} - \frac{1}{x+1} = 1$

c $\frac{2x-1}{2} - \frac{6}{x+1} = 1$

d $\frac{3}{2x-1} - \frac{4}{3x-1} = 1$

9 Simplify the following expressions.

a $\frac{x^2 + 2x - 3}{2x^2 + 7x + 3}$

b $\frac{4x^2 - 1}{2x^2 + 5x - 3}$

c $\frac{6x^2 + x - 2}{9x^2 - 4}$

d $\frac{4x^2 + x - 3}{4x^2 - 7x + 3}$

e $\frac{4x^2 - 25}{8x^2 - 22x + 5}$

Bridging GCSE and A-Level Maths

Algebraic Fractions - Answers

1 **a** $\frac{5x}{6}$ **b** $\frac{19x}{20}$ **c** $\frac{23x}{20}$ **d** $\frac{3x+2y}{6}$ **e** $\frac{x^2y+8}{4x}$

f $\frac{5x+7}{6}$ **g** $\frac{7x-3}{4}$ **h** $\frac{13x-5}{15}$ **i** $\frac{3x-1}{4}$

j $\frac{12x-23}{10}$

2 **a** $\frac{x}{6}$ **b** $\frac{11x}{20}$ **c** $\frac{7x}{20}$ **d** $\frac{3x-2y}{6}$ **e** $\frac{xy^2-8}{4y}$

f $\frac{x-1}{6}$ **g** $\frac{x-1}{4}$ **h** $\frac{-7x-5}{15}$

i $\frac{x-1}{4}$ **j** $\frac{-8x+7}{10}$

3 **a** 3 **b** 6 **c** 2 **d** 5 **e** 0.75 **f** 3

4 **a** $\frac{x^2}{6}$ **b** $\frac{3xy}{14}$ **c** $\frac{8}{3}$ **d** $\frac{2xy}{3}$ **e** $\frac{x^2-2x}{10}$

$\frac{1}{6}$ **f** **g** $\frac{6x^2+5x+1}{8}$

h $\frac{2x^2+x}{15}$ **i** $\frac{2x-4}{x-3}$ **j** $\frac{1}{2x}$

5 **a** $\frac{3}{2}$ **b** $\frac{x}{y}$ **c** $\frac{8}{3}$ **d** $\frac{2xy}{3}$ **e** $\frac{5x}{2x-4}$

f $\frac{2x^2-12x+18}{75}$ **g** 1 **h** $\frac{1}{4x+2}$

i $\frac{x^2-5x+6}{48}$ **j** $\frac{1}{2x}$

6 **a** x **b** $\frac{x}{2}$ **c** $\frac{3x^2}{16}$ **d** 3

e $\frac{17x+1}{10}$ **f** $\frac{13x+9}{10}$ **g** $\frac{3x^2-5x-2}{10}$

h $\frac{x+3}{2}$ **i** $\frac{2}{3}$ **j** $\frac{2x^2-6y^2}{9}$

8 **a** 3, −1.5 **b** 4, −1.25 **c** 3, −2.5 **d** 0, 1

i $\frac{x-1}{4}$ **j** $\frac{-8x+7}{10}$

9 **a** $\frac{x-1}{2x+1}$ **b** $\frac{2x+1}{x+3}$ **c** $\frac{2x-1}{3x-2}$

d $\frac{x+1}{x-1}$ **e** $\frac{2x+5}{4x-1}$

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Completing the Square

Another method for solving quadratic equations is **completing the square**. This method can be used to give answers to a specified number of decimal places or to leave answers in **surd** form.

You will remember that:

$$(x + a)^2 = x^2 + 2ax + a^2$$

which gives:

$$x^2 + 2ax = (x + a)^2 - a^2$$

This is the basic principle behind completing the square.

Some quadratics are **perfect squares** such as $x^2 - 8x + 16$ which can be written $(x-4)^2$. For other quadratics you can **complete the square**. This means write the quadratic in the form $(x+q)^2 + r$

Key point

The completed square form of $x^2 + bx + c$ is $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$

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Completing the Square

1 Write an equivalent expression in the form $(x \pm a)^2 - b$.

a $x^2 + 4x$ **b** $x^2 + 14x$ **c** $x^2 - 6x$ **d** $x^2 + 6x$

e $x^2 - 4x$ **f** $x^2 + 3x$ **g** $x^2 - 5x$ **h** $x^2 + x$

i $x^2 + 10x$ **j** $x^2 + 7x$ **k** $x^2 - 2x$ **l** $x^2 + 2x$

2 Write an equivalent expression in the form $(x \pm a)^2 - b$.

Question **1** will help with **a** to **h**.

a $x^2 + 4x - 1$ **b** $x^2 + 14x - 5$ **c** $x^2 - 6x + 3$ **d** $x^2 + 6x + 7$

e $x^2 - 4x - 1$ **f** $x^2 + 3x + 3$ **g** $x^2 - 5x - 5$ **h** $x^2 + x - 1$

i $x^2 + 8x - 6$ **j** $x^2 + 2x - 1$ **k** $x^2 - 2x - 7$ **l** $x^2 + 2x - 9$

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Completing the Square - Answers

1

a) $(x+2)^2-4$ b) $(x+7)^2-49$ c) $(x-3)^2-9$ d) $(x+3)^2-9$

e) $(x-2)^2-4$ f) $(x+\frac{3}{2})^2-\frac{9}{4}$ g) $(x-\frac{5}{2})^2-\frac{25}{4}$ h) $(x+\frac{1}{2})^2-\frac{1}{4}$

i) $(x+5)^2-25$ j) $(x+\frac{7}{2})^2-\frac{49}{4}$ k) $(x-1)^2-1$ l) $(x+1)^2-1$

2

a) $(x+2)^2-5$ b) $(x+7)^2-54$ c) $(x-3)^2-6$ d) $(x+3)^2-2$

e) $(x-2)^2-5$ f) $(x+\frac{3}{2})^2-\frac{21}{4}$ g) $(x-\frac{5}{2})^2-\frac{45}{4}$ h) $(x+\frac{1}{2})^2-\frac{5}{4}$

i) $(x+4)^2-24$ j) $(x+1)^2-2$ k) $(x-1)^2-8$ l) $(x+1)^2-10$

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Completing the Square

If you have an expression of the form ax^2+bx+c then first factor out the a, as shown in Example 1

Example 1

Write each of these quadratics in the form $p(x+q)^2+r$ where p, q and r are constants to be found.

a x^2+6x+7 **b** $-2x^2+12x$

a $x^2+6x+7=\left(x+\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)^2+7$

$=(x+3)^2-9+7$

$=(x+3)^2-2$

The constant term in the bracket will be half of the coefficient of x

b $-2x^2+12x=-2[x^2-6x]$

$=-2[(x-3)^2-9]$

$=-2(x-3)^2+18$

First factor out the coefficient of x^2 then complete the square for the expression in the square brackets.

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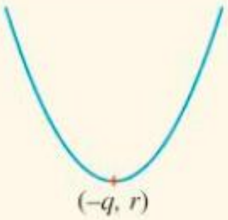
Completing the Square

Completing the square also helps us find the minimum/maximum point of a quadratic graph!

Key point

The turning point on the curve with equation $y = p(x+q)^2 + r$ has coordinates $(-q, r)$, this will be a minimum if p is positive and a maximum if p is negative.



Example 2

Find the coordinates of the turning point of the curve with equation $y = -x^2 + 5x - 2$

$$-x^2 + 5x - 2 = -\left[x^2 - 5x + 2\right]$$

First factor out the −1 then complete the square for the expression in the square brackets.

$$= -\left[\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + 2\right]$$

$$= -\left[\left(x - \frac{5}{2}\right)^2 - \frac{17}{4}\right]$$

$$= -\left(x - \frac{5}{2}\right)^2 + \frac{17}{4}$$

So the maximum point is at $\left(\frac{5}{2}, \frac{17}{4}\right)$

The curve is at its highest point when the bracket is equal to zero: $x - \frac{5}{2} = 0 \Rightarrow x = \frac{5}{2}$

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Completing the Square

1 Write each of these quadratic expressions in the form $p(x+q)^2+r$

a x^2+8x	**b** x^2-18x	**c** x^2+6x+3	**d** $x^2+12x-5$
e $x^2-7x+10$	**f** x^2+5x+9	**g** $2x^2+8x+4$	**h** $3x^2+18x-6$
i $2x^2-10x+3$	**j** $-x^2+12x-1$	**k** $-x^2+9x-3$	**l** $-2x^2+5x-1$

2 Use completing the square to find the turning point of each of these curves and state whether it is a maximum or a minimum.

a $y=x^2+14x$	**b** $y=x^2-18x+3$	**c** $y=x^2-9x$	**d** $y=-x^2+4x$
e $y=x^2+11x+30$	**f** $y=-x^2+6x-7$	**g** $y=2x^2+16x-5$	**h** $y=-3x^2+15x-2$

1 **a** $(x+4)^2-16$ **b** $(x-9)^2-81$
c $(x+3)^2-6$ **d** $(x+6)^2-41$
e $\left(x-\frac{7}{2}\right)^2-\frac{9}{4}$ **f** $\left(x+\frac{5}{2}\right)^2+\frac{11}{4}$
g $2(x+2)^2-4$ **h** $3(x+2)^2-33$
i $2\left(x-\frac{5}{2}\right)^2-\frac{19}{2}$ **j** $-(x-6)^2+35$
k $-\left(x-\frac{9}{2}\right)^2+\frac{69}{4}$ **l** $-2\left(x-\frac{5}{4}\right)^2+\frac{17}{8}$

2 **a** $(-7, -49)$ is a minimum point
b $(9, -78)$ is a minimum point
c $\left(\frac{9}{2}, -\frac{81}{4}\right)$ is a minimum point
d $(2, 4)$ is a maximum point
e $\left(-\frac{11}{2}, -\frac{1}{4}\right)$ is a minimum point
f $(3, 2)$ is a maximum point
g $(-4, -37)$ is a minimum point
h $\left(\frac{5}{2}, \frac{67}{4}\right)$ is a maximum point.

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Rearranging Formulae

Equations and formulae can be rearranged using the same method as for solving equations.

Example 3

Rearrange $Ax-3=\frac{x+B}{2}$ to make x the subject.

$2Ax-6=x+B$ — Multiply both sides by 2

$2Ax-6-x=B$ — Subtract x from both sides.

$2Ax-x=B+6$ — Add 6 to both sides.

$x(2A-1)=B+6$ — Factorise the side involving x

$x=\frac{B+6}{2A-1}$ — Divide both sides by $(2A-1)$ to make x the subject.

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Rearranging Formulae

3 Rearrange each of these formulae to make x the subject.

a $2x+5=3A-1$ **b** $x+u=vx+3$ **c** $\dfrac{3x-1}{k}=2x$ **d** $5(x-3m)=2nx-4$

e $(1-3x)^2=t$ **f** $\dfrac{1}{x}=\dfrac{1}{p}+\dfrac{1}{q}$ **g** $\dfrac{1}{x^2+k}-6=4$ **h** $\sqrt{x+A}=2B$

3 a $x=\dfrac{3A-6}{2}$ b $x=\dfrac{3-u}{1-v}$ c $x=\dfrac{1}{3-2k}$

d $x=\dfrac{15m-4}{5-2n}$ e $x=\dfrac{1\pm\sqrt{t}}{3}$ f $x=\dfrac{pq}{p+q}$

g $x=\pm\sqrt{\dfrac{1}{10}-k}$ h $x=4B^2-A$

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Simultaneous Equations

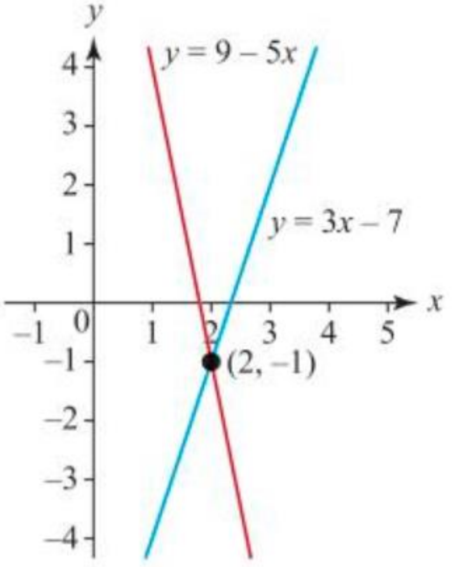
The graphs of $3x - y = 7$ and $5x + y = 9$ are shown. Only one pair of values for (x, y) satisfies both equations. This corresponds to the point of intersection of the two graphs. In this example it is $x = 2$ and $y = -1$

Equations solved together like this are called **simultaneous equations.**

Key point

You can solve a pair of linear simultaneous equations

1. Graphically.
2. By eliminating one of the **variables**.
3. By substituting an expression for one of the variables from one equation into the other.



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Simultaneous Equations - elimination

You can solve linear simultaneous equations using the **elimination** method, as shown in Example 4. The solutions to simultaneous equations give the point of intersection between the lines represented by the two equations.

Example 4

Solve the simultaneous equations $5x-4y=17$, $3x+8y=5$

$15x+40y=25 \quad (1)$ — Multiply the second equation by 5

$15x-12y=51 \quad (2)$ — Multiply the first equation by 3

$(1)-(2): 52y=-26$ — Subtract equation (2) from equation (1) to eliminate x

$y=-\frac{1}{2}$

$5x-4\left(-\frac{1}{2}\right)=17$ — Substitute $y=-\frac{1}{2}$ into one of the original equations.

$5x+2=17$ — Solve this equation to find the value of x

$5x=15$

$x=3$

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Simultaneous Equations - elimination

4 Use algebra to solve each of these pairs of simultaneous equations.

a $5x+12y=-6,\ \ x+5y=4$ **b** $7x+5y=14,\ \ 3x+4y=19$ **c** $2x-5y=4,\ \ 3x-8y=5$

d $3x-2y=2,\ \ 8x+3y=4.5$ **e** $5x-2y=11,\ \ -2x+3y=22$ **f** $8x+5y=-0.5,\ \ -6x+4y=-3.5$

4 a $y=2, x=-6$ **b** $x=-3, y=7$ **c** $y=2, x=7$

d $=\frac{3}{5}, y=-\frac{1}{10}$ **e** $y=12, x=7$ **f** $y=-\frac{1}{2}, x=\frac{1}{4}$

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Simultaneous Equations - substitution

Solve the simultaneous equations $y = 2x + 3$, $3x + 4y = 1$.

Because the first equation is in the form $y = \ldots$ it suggests that the substitution method should be used.

Equations should still be labelled to help with explaining the method

$y = 2x + 3$ (1)

$3x + 4y = 1$ (2)

Step 1: As equation (1) is in the form $y = \ldots$ there is no need to rearrange an equation.

Step 2: Substitute the right hand side of equation (1) into equation (2) for the variable y.

$$3x + 4(2x + 3) = 1$$

Step 3: Expand and solve the equation. $3x + 8x + 12 = 1$, $11x = -11$, $x = -1$

Step 4: Substitute $x = -1$ into $y = 2x + 3$: $y = -2 + 3 = 1$

Step 5: Test the values in $y = 2x + 3$ which gives $1 = -2 + 3$ and $3x + 4y = 1$, which gives $-3 + 4 = 1$. These are correct so the solution is $x = -1$ and $y = 1$.

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Simultaneous Equations - substitution

Solve the simultaneous equations $3x + y = 5$ and $5x - 2y = 12$ using the substitution method.

Looking at the equations there is only one that could be sensibly rearranged without involving fractions.

Step 1: Rearrange $3x + y = 5$ to get $y = 5 - 3x$ (1)

Step 2: Substitute $y = 5 - 3x$ into $5x - 2y = 12$ (2)

$$5x - 2(5 - 3x) = 12$$

Step 3: Expand and solve $5x - 10 + 6x = 12 \Rightarrow 11x = 22 \Rightarrow x = 2$

Step 4: Substitute into equation (1): $y = 5 - 3 \times 2 = 5 - 6 = -1$

Step 5: Check: (1), $3 \times 2 + -1 = 5$ and (2), $5 \times 2 - 2 \times -1 = 10 + 2 = 12$, which are correct so the solution is $x = 2$ and $y = -1$.

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Simultaneous Equations – non-linear

We can use a similar method when we need to solve a pair of equations, one of which is linear and the other of which is **non-linear**. But we must *always* **substitute** from the linear into the non-linear.

Solve these simultaneous equations.

$$x^2 + y^2 = 5$$

$$x + y = 3$$

Call the equations (1) and (2):

$$x^2 + y^2 = 5 \quad (1)$$

$$x + y = 3 \quad (2)$$

Rearrange equation (2) to obtain:

$$x = 3 - y$$

Substitute this into equation (1), which gives:

$$(3 - y)^2 + y^2 = 5$$

Expand and rearrange into the general form of the quadratic equation:

$$9 - 6y + y^2 + y^2 = 5$$

$$2y^2 - 6y + 4 = 0$$

Cancel by 2:

$$y^2 - 3y + 2 = 0$$

Factorise:

$$(y - 1)(y - 2) = 0$$

$$\Rightarrow \quad y = 1 \text{ or } 2$$

Substitute for y in equation (2):

When $y = 1$, $x = 2$; and when $y = 2$, $x = 1$.

Note you should always give answers as a pair of values in x and y.

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Simultaneous Equations - substitution

Solve the following equations

a)

$$a + 2b = 9$$
$$a + 3b = 11$$

b)

$$2x + 5y = 46$$
$$x + 3y = 27$$

c)

$$3m + 2n = 14$$
$$4m + 5n = 14$$

d)

$$p + r = 15$$
$$p - r = 9$$

e)

$$3a + 2b = 33$$
$$5a - 4b = 44$$

f)

$$6x - 5y = 20$$
$$4x + 3y = 7$$

g)

$$2m + 5n = 57$$
$$n = m + 2$$

h)

$$4p + 3r = 5$$
$$r = 2p - 5$$

i)

$$a - 5b = 6$$
$$a = 3b + 2$$

j)

$$x^2 + y^2 = 45$$
$$x = 2y$$

k)

$$x^2 - y^2 = -5$$
$$y = x - 5$$

l)

$$2x^2 + y^2 = 57$$
$$x + y = 1$$

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Simultaneous Equations - substitution

a) a = 5, b = 2	b) x = 3, y = 8	c) m = 6, n = -2
d) p = 12, r = 3	e) a = 10, b = 1.5	f) x = 2.5, y = -1
g) $m = 4\frac{1}{7}, n = 2\frac{1}{7}$	h) p = 2, r = -1	i) a = -4, b = -2
j) x = 6, y = 3 x = -6, y = -3	k) x = 2, y = -3	l) x = -4, y = 5

Bridging GCSE and A-Level Maths

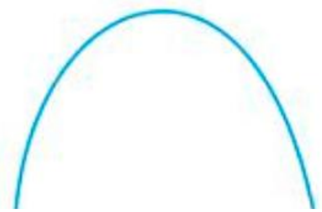
Drawing Graphs - Quadratics

A **quadratic function** can be written in the form $ax^2 + bx + c$, where a, b and c are constants and $a \neq 0$

Quadratic curves are symmetrical about their **vertex** (the turning point). For $a > 0$, this vertex is always a **minimum** point, and for $a < 0$ this vertex is always a **maximum** point.

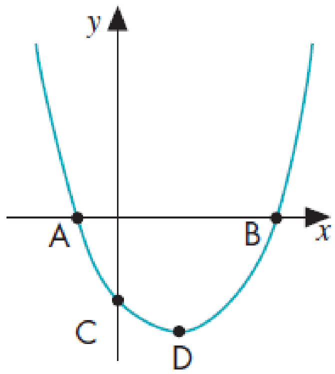
When $a > 0$, a quadratic graph looks like this.

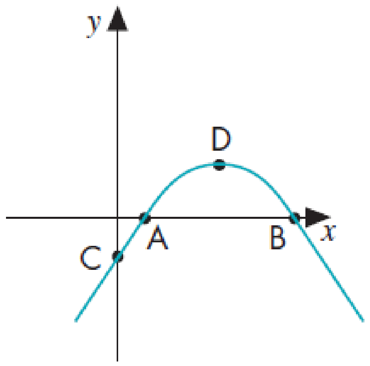
When $a < 0$, a quadratic graph looks like this.



The significant points of a quadratic graph

A quadratic graph has four points that are of interest to a mathematician. These are the points A, B, C and D on the diagram. A and B are called the **roots**, and are where the graph crosses the x-axis, C is the point where the graph crosses the y-axis (the **intercept**) and D is the **vertex**, and is the lowest or highest point of the graph.





The roots

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Drawing Graphs - Quadratics

The roots

These are the points where the graph crosses the x-axis therefore $y = 0$ at these points.
You can find the roots by solving the quadratic written in the from $ax^2 + bx + c = 0$.
You can solve by factorisation, using the quadratic formula or completing the square.

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Drawing Graphs - Quadratics

For each quadratic function

- **i** Factorise the equation,
- **ii** Use your answer to part **i** to sketch a graph of the function.

a $f(x) = x^2 + 3x + 2$ **b** $f(x) = x^2 + 6x - 7$

c $f(x) = -x^2 - x + 2$ **d** $f(x) = -x^2 - 7x - 12$

e $f(x) = 2x^2 - x - 1$ **f** $f(x) = -3x^2 + 11x + 20$

Hint:
Remember to include the y-intercept. This is when x = 0 so you can substitute that in to your equation.

You can leave out the vertex for now.

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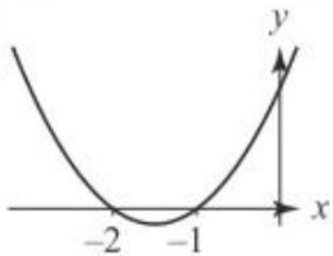
Drawing Graphs - Quadratics

2 **a** **i** $f(x) = x^2 + 3x + 2 = (x + 1)(x + 2)$

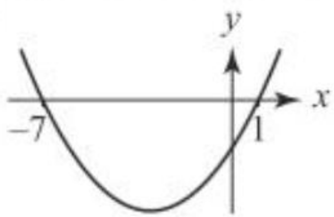
ii



y-intercept = 2

b **i** $f(x) = x^2 + 6x - 7 = (x - 1)(x + 7)$

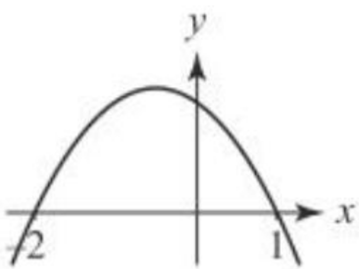
ii



y-intercept = -7

c **i** $f(x) = -x^2 - x + 2 = (x + 2)(1 - x)$

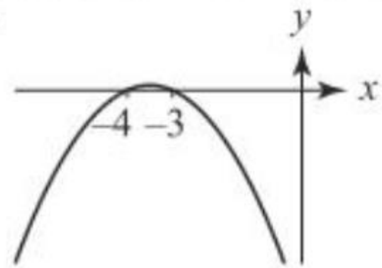
ii



y-intercept = 2

d **i** $f(x) = -x^2 - 7x - 12 = (x + 4)(-x - 3)$

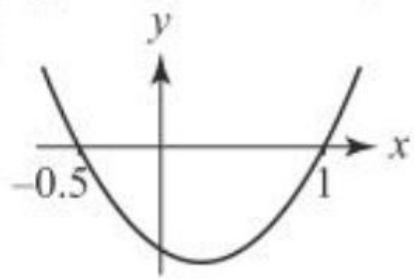
ii



y-intercept = -12

e **i** $f(x) = 2x^2 - x - 1 = (2x + 1)(x - 1)$

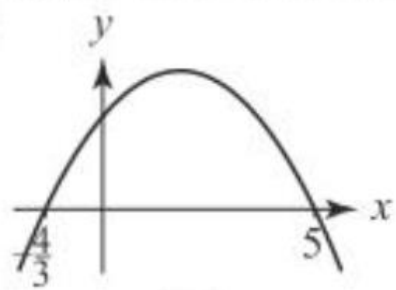
ii



y-intercept = -1

f **i** $f(x) = -3x^2 + 11x + 20 = (x - 5)(-3x - 4)$

ii



y-intercept = 20

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Drawing Graphs - Cubics

Cubic graphs

A **cubic** function or graph is one which contains a term in x^3. The following are examples of cubic graphs:

$y = x^3$ $\quad$ $y = x^3 - 2x^2 - 3x - 4$ $\quad$ $y = x^3 - x^2 - 4x + 4$ $\quad$ $y = x^3 + 3x$

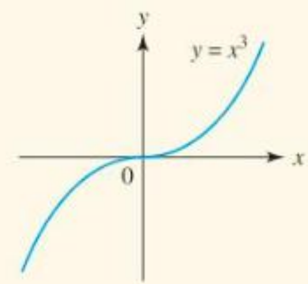
Key point

A sketch should always show:

- The shape of the curve.
- The position of the curve on the axes.
- Points of intersection with the x and y axes.

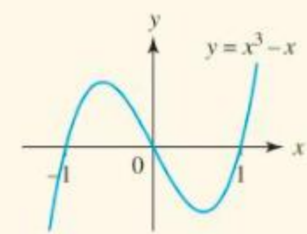
If the question calls for it, a sketch may also include:

- Maximum and minimum points.
- The location of a named point.



The graph of $y=x^3$ is shown. It crosses through the x-axis once only, at the point (0, 0), since the equation $x^3=0$ has just one real solution $x=0$

A cubic equation can have three real solutions, in which case it will cross the x-axis three times. For example, the equation $y=x^3-x$ crosses the x-axis at $x=-1$, $x=0$ and $x=1$ since the equation $x^3-x=0$ can be solved by factorising in this way.

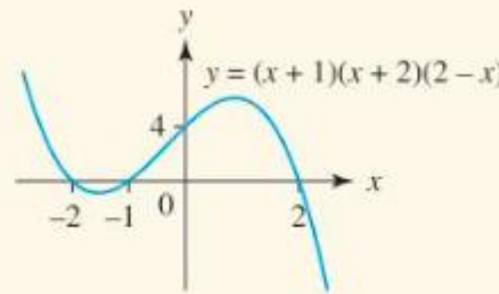


$$x^3-x=0 \Rightarrow x(x^2-1)=0$$
$$\Rightarrow x(x-1)(x+1)=0$$
$$\Rightarrow x=0,\ 1,\ -1$$

Bridging GCSE and A-Level Maths

Drawing Graphs - Cubics

A cubic equation will look different if the coefficient of the x^3 term is negative. For example, the equation $y=(x+1)(x+2)(2-x)$ crosses the x-axis at $x=-1$, $x=-2$ and $x=2$, but if you expand $(x+1)(x+2)(2-x)$ you get $-x^3-x^2+4x+4$ so the coefficient of the x^3 term is negative.



You can also work out where a graph crosses the y-axis by substituting $x=0$ into the equation, in this case at $y=4$

Example 1

Sketch the graph with equation $y=x(x+4)(5-x)$

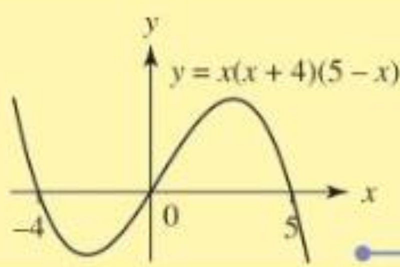
The equation $y=x(x+4)(5-x)$ crosses the x-axis at $x=0$, $x=-4$ and $x=5$

Since these are the solutions to $x(x+4)(5-x)=0$

Expanding $x(x+4)(5-x)$ gives $-x^3+x^2+20x$

You don't necessarily need to expand the whole expression to work out the coefficient of x^3

So the coefficient of x^3 is negative.

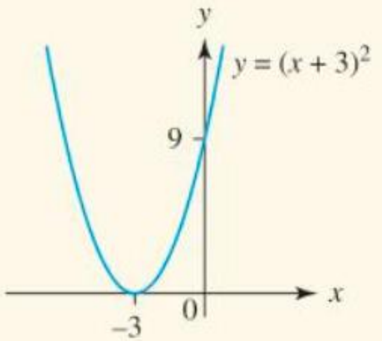


Sketch the graph. Remember to label the intercepts.

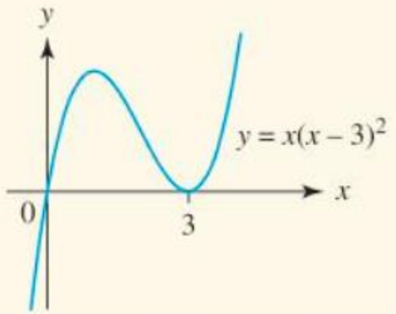
Bridging GCSE and A-Level Maths

Drawing Graphs - Cubics

Sometimes a polynomial equation will have **repeated roots**. A repeated root occurs when the polynomial has a squared factor, so the same root is given twice. This means the x axis is a tangent to the curve at the root. For example, the quadratic equation $x^2+6x+9=0$ can be factorised and written as $(x+3)^2=0$, which means that there is a repeated root of $x=-3$. So the graph of $y=x^2+6x+9$ will just touch the x-axis at the point $x=-3$ as shown.



The same principle applies to cubic equations. For example the equation $y=x(x-3)^2$ crosses the x-axis at $x=0$ and touches the x-axis at $x=3$ because this is a repeated root.

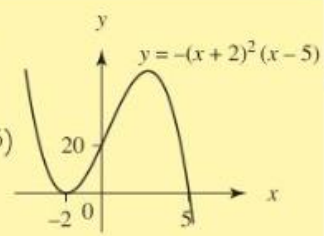


Example 2

Sketch the graph with equation $y=-(x+2)^2(x-5)$

$-(x+2)^2(x-5)=0$ has a root at $x=5$ and a repeated root at $x=-2$

Therefore the graph of $y=-(x+2)^2(x-5)$ will cross the x-axis at $x=5$ and touch at $x=-2$



The coefficient of x^3 is negative so the graph is this way around.

When $x=0$, $y=-(2)^2(-5)=20$

Find the point of intersection with the y-axis.

Bridging GCSE and A-Level Maths

Drawing Graphs - Cubics

1 Sketch each of these cubic graphs.

a $y = -x^3$

b $y = (x+1)(x+2)(x+4)$

c $y = (x-2)(x+3)(x+5)$

d $y = x(x+1)(x-2)$

e $y = (5-x)(x+2)(x+6)$

f $y = -x(x+1)(x-7)$

g $y = x^2(x+3)$

h $y = (x-1)^2(x+4)$

i $y = -x(x+5)^2$

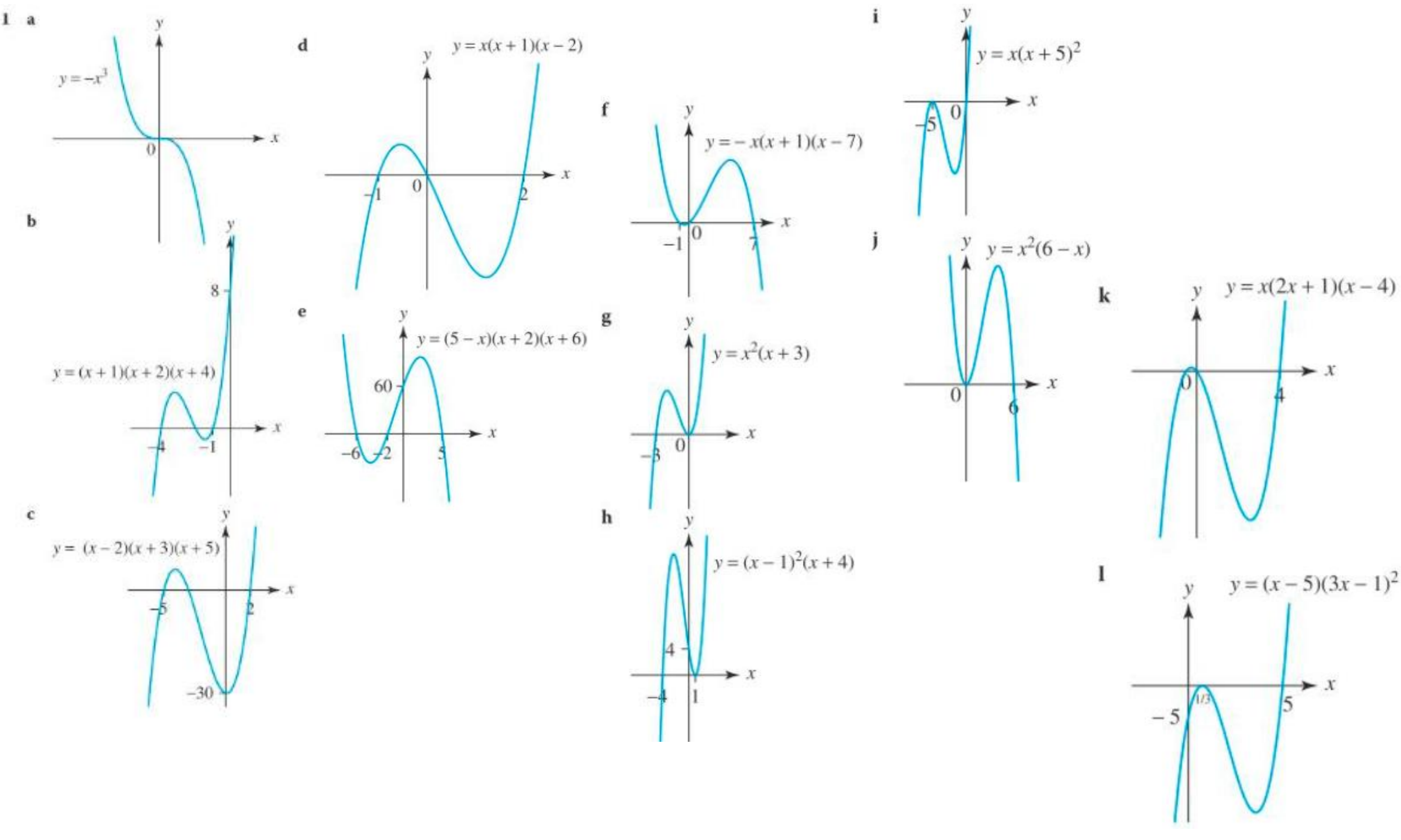
j $y = x^2(6-x)$

k $y = x(2x+1)(x-4)$

l $y = (x-5)(3x-1)^2$

Bridging GCSE and A-Level Maths

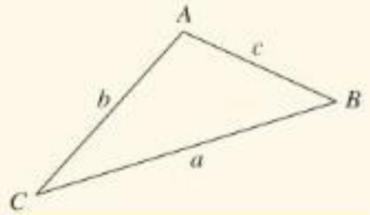
Drawing Graphs - Cubics



Bridging GCSE and A-Level Maths

Trigonometry – Sine Rule

To solve problems involving non-right-angled triangles you use the **sine** rule and the **cosine** rule.



Key point

Sine rule: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ to find an angle,

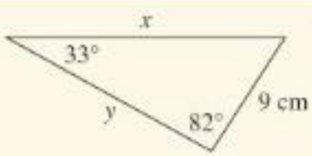
or write as $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ to find a side. It's important that sides and angles with the same letter are opposite.

Angles are written upper case and sides are lower case. Angle A is opposite side a and so on.

In order to use the sine rule you must have information about an opposite side and angle pair.

Example 1

Find the lengths of sides x and y in this triangle.



$$\frac{x}{\sin 82} = \frac{9}{\sin 33}$$

The side of length 9 cm is opposite the 33° angle. The side x is opposite the 82° angle. So you can use the sine rule.

$$x = \frac{9}{\sin 33} \times \sin 82 \quad = 16.4 \text{ cm}$$

Rearrange to solve for x

The angle opposite y is $180 - 33 - 82 = 65°$

$$\text{So } \frac{y}{\sin 65} = \frac{9}{\sin 33}$$

You could also use the other pair of x and 82°: $\frac{y}{\sin 65} = \frac{16.4}{\sin 82}$

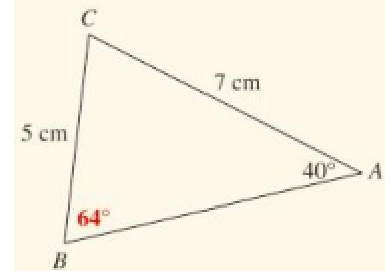
$$y = \frac{9}{\sin 33} \times \sin 65 \quad = 15.0 \text{ cm}$$

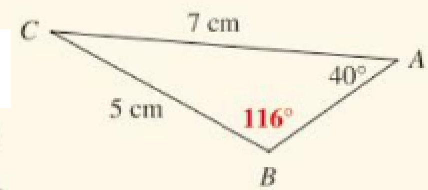
Bridging GCSE and A-Level Maths

Trigonometry – Sine Rule

When finding an angle, remember that the equation $\sin\theta = k$ has two solutions in the range $0° < \theta < 180°$ when $0 < k < 1$. So you need to subtract the first solution you find from 180° to find a second solution then decide whether or not it is a possible solution for your triangle.

For example, if $A = 40°$, $a = 5$ and $b = 7$ then two different triangles could be formed:



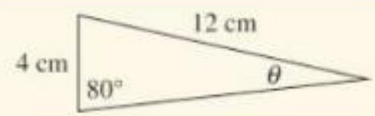


The sine rule gives the acute solution $B = 64°$, but $B = 180 - 64 = 116°$ is also a possible solution.

You need to check whether or not the obtuse solution will actually work. If it is too big then the angle sum of the triangle would be more than 180° which is not possible!

Example 2

Find the size of angle θ in this triangle.



$$\frac{\sin\theta}{4} = \frac{\sin 82}{12}$$

The side of length 12 cm is opposite the 80° angle so you can use the sine rule. The angle θ is opposite a side of length 4 cm.

$$\sin\theta = \frac{\sin 80}{12} \times 4 = 0.328...$$

Rearrange to solve for θ

$$\theta = \sin^{-1}(0.328...) = 19.2°$$

$$\text{or } \theta = 180 - 19.2 = 160.8°$$

Subtract from 180° to give other solution.

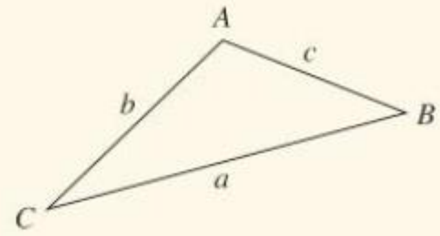
but 160.8° is not possible for this triangle as it would give an angle sum of more than 180°

So, $\theta = 19.2°$

Bridging GCSE and A-Level Maths

Trigonometry – Cosine Rule

If you do not have information about an opposite side and angle pair then you will need to use the cosine rule.



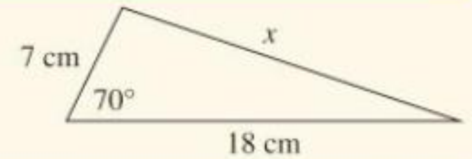
Key point

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

(where angle A is opposite side a)

Example 3

Find the length of side x in this triangle.



$x^2 = 7^2 + 18^2 - 2 \times 7 \times 18 \times \cos 70$

$= 286.8$

$x = \sqrt{286.8} = 16.9$ cm

You do not have information about an opposite side and angle pair so you need to use the cosine rule.

x is the side you need to find so this is 'a' in the rule, which means the 70° angle is 'A' since it is opposite x. The other two sides are 'b' and 'c' in either order.

Bridging GCSE and A-Level Maths

Trigonometry – Cosine Rule

If you know the lengths of all three sides of a triangle then you can use the cosine rule to find one of the angles. You can either use $a^2 = b^2 + c^2 - 2bc\cos A$ and solve to find A, or you can use the rearranged version of the cosine rule:

Cosine rule: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

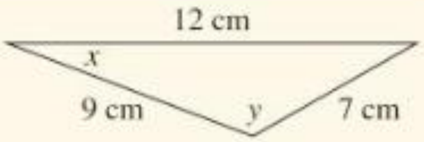
(Remember that side a is opposite angle A)

The equation $\cos\theta = k$ only has one solution in the range $0° < \theta < 180°$ so the cosine rule gives a unique solution.

You can also use trigonometry to find the area of a triangle.

Example 4

Find the size of angles x and y in this triangle.



$$\cos x = \frac{(12^2 + 9^2 - 7^2)}{2 \times 9 \times 12} = \frac{22}{27}$$

$$x = \cos^{-1}\left(\frac{22}{27}\right) = 35.4°$$

You know the lengths of all three sides so use the cosine rule.

$$\cos y = \frac{9^2 + 7^2 - 12^2}{2 \times 9 \times 7} = -\frac{1}{9}$$

$$y = \cos^{-1}\left(-\frac{1}{9}\right) = 96.4°$$

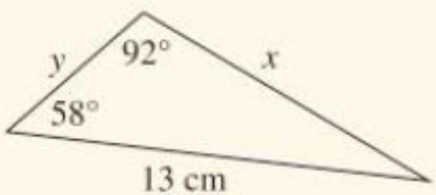
Make sure you use 12 as the side opposite to angle y

Bridging GCSE and A-Level Maths

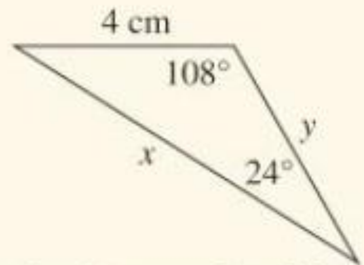
Trigonometry

1 Use the sine rule to find the length of the sides labelled x and y in each of these triangles.

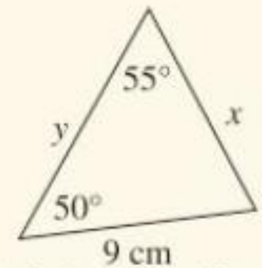
a



b

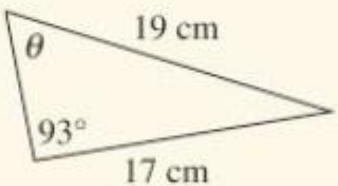


c

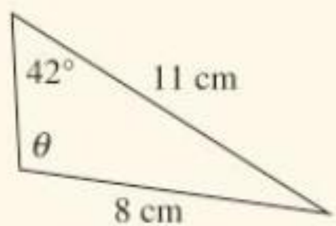


2 Use the sine rule to find the size of angle θ in each of these triangles. Explain whether the solution is unique in each case.

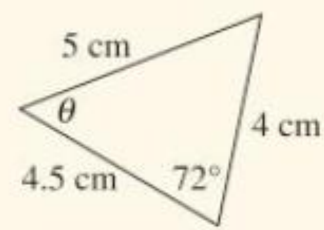
a



b

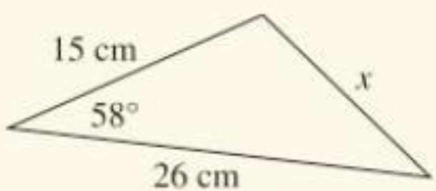


c

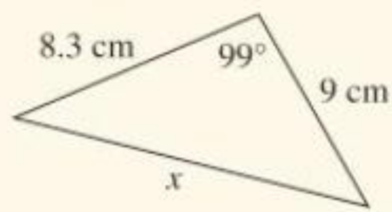


3 Use the cosine rule to find the length of the side x in each of these triangles.

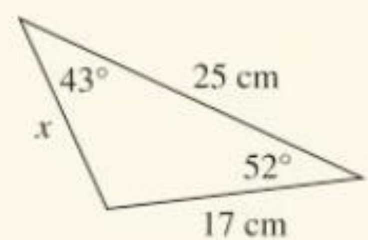
a



b



c

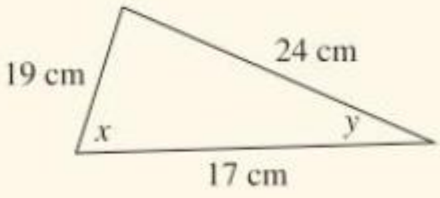


Bridging GCSE and A-Level Maths

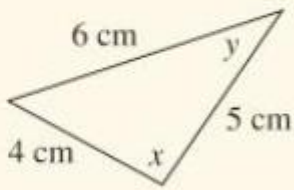
Trigonometry

4 Use the cosine rule to find the size of the angles labelled x and y in each of these triangles.

a



b



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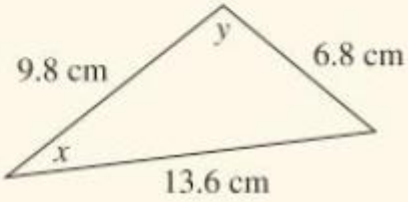
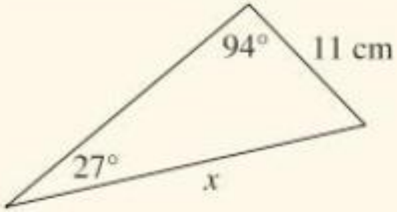


5 Find the length of the side labelled x in each of these triangles.

a



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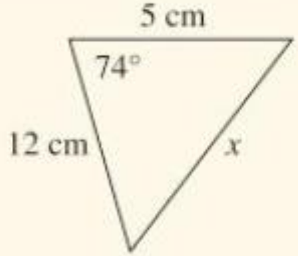

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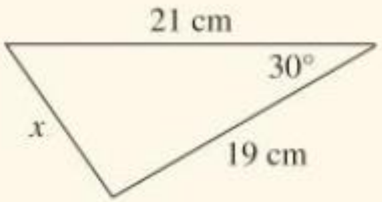

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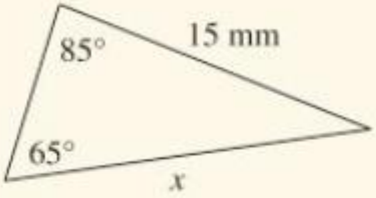

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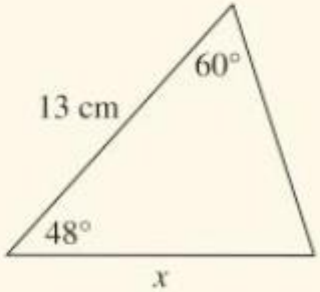

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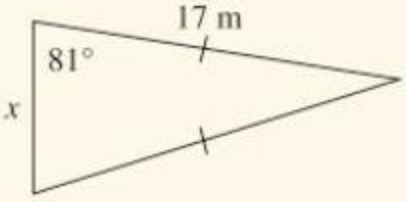
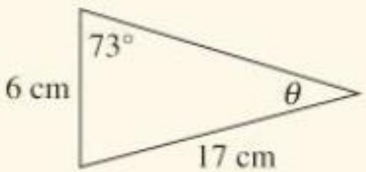


Bridging GCSE and A-Level Maths

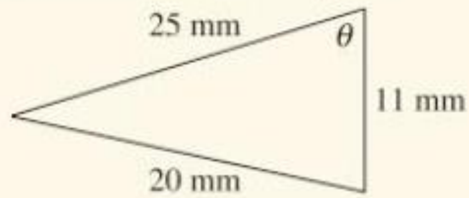
Trigonometry

6 Find the size of the acute angle θ in each of these triangles.

a



b



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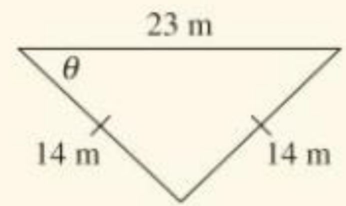

7 Triangle ABC is such that $AB = 5$ cm, $BC = 3$ cm and $AC = 7$ cm. Calculate the size of angle ABC

8 Triangle ABC is such that $AB = 24$ cm, $AC = 27$ cm, angle $ABC = 37°$ and angle $BCA = \theta$. Calculate θ

1 a $x = 11.0$ cm, $y = 6.50$ cm
b $x = 9.35$ cm, $y = 7.31$ cm
c $x = 8.42$ cm, $y = 10.6$ cm

2 a $\theta = 63.3°$
116.7° is not possible since the angle sum would be more than 180°, so the answer is unique.
b $\theta = 66.9°$
113.1° is a possible solution so the answer is not unique.
c $\theta = 49.5°$
130.5° is not possible since the angle sum would be more than 180°, so the answer is unique.

3 a 22.1 cm b 13.2 cm c 19.8 cm

4 a $x = 83.4°$, $y = 51.9°$
b $x = 82.8°$, $y = 41.4°$
c $x = 28.3°$, $y = 108.7°$

5 a 24.2 cm b 11.7 cm c 10.5 cm
d 16.5 mm e 11.8 cm f 5.32 m

6 a 19.7° b 51.0° c 34.8°

7 120°

8 32.3°